VOLUME LI NUMBER 4

GENERAL LIBRARY
JUN 21 1920
UNIV. OF MICH.

THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

EDITED BY

GEORGE E. HALE
Mount Wilson Observatory of the Carnegie Institution of Washington

EDWIN B. FROST
Yerkes Observatory of the University of Chicago

HENRY G. GALE
Ryerson Physical Laboratory of the University of Chicago

MAY 1920

THE ECLIPSING VARIABLE STAR, λ TAURI . . . . . *Joel Stebbins* 193

THE ELLIPSOIDAL VARIABLE STAR, 19 ORIONIS . . . . . *Joel Stebbins* 210

A MODIFICATION OF THE ELECTRON THEORY OF DISPERSION TO ACCOUNT FOR THE CHANGE OF THE REFRACTIVE INDEX WITH TEMPERATURE . . . . *E. O. Hulburt* 223

FINE STRUCTURE OF THE NEAR INFRA-RED ABSORPTION BANDS OF THE HALOGEN ACIDS . . . . . *Walter F. Colby* 230

NOTE ON THE AIR LINES IN SPARK SPECTRA FROM λ 5927 TO λ 8719 . . . *Paul W. Merrill* 236

THE GOLD-POINT PALLADIUM-POINT BRIGHTNESS RATIO . . *Edward P. Hyde and W. E. Forsythe* 244

MINOR CONTRIBUTIONS AND NOTES:
Five O5 Stars with Variable Radial Velocities, W. CARL RUFUS, 252; *The Parallax of the B-Star Boss 1517*, J. VOÛTE, 254.

PREPARATION OF ABSTRACTS 253

THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILLINOIS, U.S.A.

THE CAMBRIDGE UNIVERSITY PRESS, London
THE MARUZEN-KABUSHIKI-KAISHA, Tokyo, Osaka, Kyoto, Fukuoka, Sendai
THE MISSION BOOK COMPANY, Shanghai

May 1920 Vol. LI, No. 4

THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

EDITED BY

GEORGE E. HALE
Mount Wilson Observatory of the Carnegie Institution of Washington

EDWIN B. FROST
Yerkes Observatory of the University of Chicago

HENRY G. GALE
Ryerson Physical Laboratory of the University of Chicago

WITH THE COLLABORATION OF

JOSEPH S. AMES, Johns Hopkins University
ARISTARCH BELOPOLSKY, Observatoire de Poulkova
WILLIAM W. CAMPBELL, Lick Observatory
HENRY CREW, Northwestern University
CHARLES FABRY, Université de Marseille
ALFRED FOWLER, Imperial College, London
CHARLES S. HASTINGS, Yale University
HEINRICH KAYSER, Universität Bonn
ALBERT A. MICHELSON, University of Chicago
HUGH F. NEWALL, Cambridge University
ERNEST F. NICHOLS, Yale University
ALFRED PEROT, Paris
CARL RUNGE, Universität Göttingen
HENRY N. RUSSELL, Princeton University
SIR ARTHUR SCHUSTER, Twyford
FRANK SCHLESINGER, Yale Observatory

The Astrophysical Journal is published by the University of Chicago at the University of Chicago Press, 5750 Ellis Avenue, Chicago, Illinois, during each month except February and August. ¶The subscription price is $5.00 a year; the price of single copies is 65 cents. Orders for service of less than a half-year will be charged at the single-copy rate. ¶Postage is prepaid by the publishers on all orders from the United States, Mexico, Cuba, Porto Rico, Panama Canal Zone, Republic of Panama, Hawaiian Islands, Philippine Islands, Guam, Samoan Islands, Shanghai. ¶Postage is charged extra as follows: For Canada, 30 cents on annual subscriptions (total $5.30), on single copies, 3 cents (total 68 cents); for all other countries in the Postal Union, 62 cents on annual subscriptions (total $5.62), on single copies, 11 cents (total 76 cents). ¶Patrons are requested to make all remittances payable to The University of Chicago Press in postal or express money orders or bank drafts.

The following are authorized to quote the prices indicated:

For the British Empire: The Cambridge University Press, Fetter Lane, London, E.C. 4. Yearly subscriptions, including postage, 26s. each; single copies, including postage, 3s. 9d. each.

For Japan and Korea: The Maruzen-Kabushiki-Kaisha, 11 to 16 Nihonbashi Tori Sanchome, Tokyo, Japan. Yearly subscriptions, including postage, Yen 11.25 each; single copies, including postage, Yen 1.30 each.

For China: The Mission Book Company, 13 North Szechuen Road, Shanghai. Yearly subscriptions, $5.00; single copies, 65 cents, or their equivalents in Chinese money. Postage extra, if mailed direct outside of Shanghai, on yearly subscriptions 62 cents, on single copies 11 cents.

Claims for missing numbers should be made within the month following the regular month of publication. The publishers expect to supply missing numbers free only when losses have been sustained in transit, and when the reserve stock will permit.

Business correspondence should be addressed to The University of Chicago Press, Chicago, Illinois.

Communications for the editors and manuscripts should be addressed to the Editors of THE ASTROPHYSICAL JOURNAL, Yerkes Observatory, Williams Bay, Wisconsin.

The cable address is "University, Chicago."

The articles in this Journal are indexed in the *Readers' Guide Supplement*, New York, N.Y.

Entered as second-class matter, January 17, 1895, at the Post-office at Chicago, Ill., under the act of March 3, 1879.

Acceptance for mailing at special rate of postage provided for in Section 1103, Act of October 3, 1917, authorized on July 15, 1918.

THE
ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME LI MAY 1920 NUMBER 4

THE ECLIPSING VARIABLE STAR, λ TAURI

By JOEL STEBBINS

ABSTRACT

Eclipsing variable star, λ Tauri; photometric study.—Two series of observations made with a sensitive photo-electric photometer during 1916–1917 and 1917–1918 have enabled the *light-curve* of this third-magnitude star to be determined (Fig. 1, Table IX). It is found to be a normal eclipsing system with two minima and continuous variation between eclipses. A comparison of the times of minima with those computed from spectroscopic studies shows a satisfactory agreement. *No light effect due to the third body*, whose existence was shown by Schlesinger to account for the observed spectroscopic orbit, is clearly evident, although a few observations, discordant up to $0^{M}.1$, occurred at such dates as to suggest they might be caused by this body. Because of the undetermined light of the third body and because of the small range of about $0^{M}.4$ in a partial eclipse, *the elements for this system* can be determined, as yet, only roughly. Plausible values for the *sizes*, *densities*, and relative *brightness* of the bodies, and for the *inclination* and *ellipsoidal constants*, are given in Table X and Fig. 3. The *difference in brightness of opposite sides of the second body* is probably chiefly due to the heating effect of the intense radiation from the first body. The author calls attention to a systematic discrepancy of $0^{M}.01$ between the two series of observations of 1916–1917 and 1917–1918, which is unexplained.

Stars of Class B, π⁴ Orionis, ξ Tauri, e Tauri, μ Tauri.—These stars were used for comparison in the above series of photo-electric measurements and none was observed to vary by as much as $0^{M}.05$, although variables are common in this class.

Photo-electric photometer for stellar observations.—After a large number of trials, a potassium photo-electric cell of fused quartz has been developed which makes the photometer much more sensitive than the former selenium photometer and which enables sixth-magnitude stars to be studied with a 12-inch telescope.

The variability of λ Tauri was established by Baxendell[1] in 1848, this being the third discovery of an eclipsing star, following

[1] *Monthly Notices*, 9, 37, 1848.

Algol and S Cancri, while with a magnitude of 3.3 λ Tauri ranks in brightness next to Algol among the older known stars of this class. Although it was immediately found that λ Tauri has a period of 3.95 days, there has been no good determination of the light-curve in the sixty years since the discovery of the variation, observers recording irregularities in the times of minima amounting to several hours. Apparently because of the small range of variation and few suitable comparison stars this variable is unusually difficult for visual estimates.

In 1897 Belopolsky found λ Tauri to be a spectroscopic binary, and the orbit has since been well determined at Allegheny. Schlesinger[1] has shown that the anomalies in the orbital motion can be explained by the presence of a third body which revolves about the eclipsing system in a period of 34.6 days. He was unable to detect the spectrum of more than one component on the plates, though there were certain peculiarities in the lines. The third body might account for large disturbances of the four-day eclipsing pair, though the spectroscopic results for the short-period orbit show no great discrepancies.

When the study of Algol with the selenium photometer was completed in 1910, the next bright eclipsing star available was λ Tauri, but the change to a whole magnitude fainter than Algol made this star, especially at minimum, entirely too faint for satisfactory observation, and attempts at measures were postponed until the apparatus could be improved. After several years the selenium photometer was discarded in favor of the new photo-electric cell made in our physics laboratory by Dr. Jakob Kunz and myself, and finally in 1916 the photo-electric photometer was in such a stage of development that stars considerably fainter than λ Tauri could be observed. A comparison of the relative performances of the selenium and photo-electric instruments is somewhat difficult, but it is safe to say that with the new device, attached to the same 12-inch refractor, stars at least three magnitudes fainter can be observed than with the selenium photometer. In fact, a careful comparison might show a difference of five magnitudes, or a one-hundred fold improvement, as the present

[1] *Publications of the Allegheny Observatory*, **3**, 167, 1915.

measures of fifth-magnitude stars are better than the measures of any stars whatever with selenium. The present practical limit is about magnitude 6.0 for stars of spectrum class A. The "color equation" of the installation, that is, the measured difference of two stars of classes A and K of the same visual brightness, is $0^{M}.86$.

I hope sometime to publish a detailed description of the photometer and the method of observing with it. No great changes have been introduced in the instrument since the work in 1915 on β Lyrae,[1] but the one important new feature is the potassium photo-electric cell which is of fused quartz. This substitution of quartz for glass in the cell wall has eliminated most of the so-called "dark current," and the particular cell now in use, our number QK99, is far superior to any cell which we had previously made. Its number indicates that it was the ninety-ninth cell which was enough of a success to be given a number, failures not being counted.

The observations of λ Tauri were begun here in October, 1916, using ξ Tauri as the principal comparison star, but also π^4 Orionis and π^5 Orionis. It was not long before the discrepancies indicated that π^5 Orionis is a variable, going through its changes in the spectroscopic period of 3.70 days, and the results for this star are given in the following article in this *Journal*. It was not possible to secure enough observations for a definitive light-curve of λ Tauri during the first season, but the main characteristics of the variation were established, there being primary and secondary eclipses as expected, and also continuous variation between minima, due to ellipticity of the components and to radiation effect. Toward the end of the season it was suspected that certain anomalies were repeated in the period of 34.6 days, showing an effect of the third body, and it was therefore determined to observe the star carefully the following year. In the meantime we had begun the practice of using neutral shade glasses to cut down the light of bright stars, and two fainter comparison stars were added, *e* Tauri and μ Tauri. In these observations the light of λ Tauri was reduced to approximate equality with the comparison star. Only one of the five comparison stars, *e* Tauri, is

[1] *Lick Observatory Bulletin*, 8, 186, 1916.

not in the list of spectroscopic binaries, and it is just among these stars of class B that variables are to be expected, but I have succeeded in establishing a light-curve only in the case of π^5 Orionis.

TABLE I

COMPARISON STARS

H.R.	Star	Spectroscopic Period	Spectrum	Visual Magnitude	Shade Used on λ Tauri	Photo-electric Reduction to ξ Tauri
1239....	λ Tauri	$3^d.952941$	B3	Var.	.....	.....
1038....	ξ Tauri	Unknown	B8	3.75	.....	.....
1174....	ε Tauri	.....	B3	5.03	$1^M.56$	$0^M.215$
1320....	μ Tauri	Unknown	B5	4.32	0.86	0.250
1552....	π^4 Orionis	9.5191	B3	3.78	.....	0.120
1567....	π^5 Orionis	3.70045	B3	3.87	.....	Var.

In Table II are the observations of λ Tauri made with the photo-electric photometer. The first date of Series I is October 22, 1916, and of Series II, September 10, 1917. The phase in the second column is computed from the elements:

$$\text{Minimum} = \text{J.D. } 2421194.573 + 3^d.952941 \cdot E,$$

the period being that of v. Aretin.[1] The phase for the monthly period is based upon the hypothetical eclipse times from Schlesinger's elements:

$$\text{Minimum} = \text{J.D. } 2417831.30 + 34^d.60 \cdot E.$$

In computing the phases the times were reduced to the sun. The difference of magnitude is referred to ξ Tauri as standard, using the corrections for the other comparison stars as given in the last column of Table I, except in the case of π^5 Orionis, the corrections for which are in Table III. Each magnitude is usually the mean of three sets of measures, a set consisting of two measures of the comparison star, four of the variable, and then two of the comparison star, the rate of observing being about six sets per hour. The residuals were taken in part graphically from the final light-

[1] *Vierteljahrsschrift der Astronomischen Gesellschaft*, **48**, 365, 1913.

TABLE II

OBSERVATIONS OF λ TAURI

SERIES I

Date, G.M.T.	$3^d.95$ Phase	$34^d.46$ Phase	Difference of Magnitude	Residual	Comparison Star	Remarks
2421159.776..	$0^d.785$	$6^d.88$	$0^M.334$	$-0^M.003$	ξ	
.810..	0.819	6.92	.347	+ .007	ξ	
1163.733..	0.789	10.84	.329	− .008	ξ	
.756..	0.812	10.86	.327	− .012	ξ	
1164.696..	1.754	11.80	.320	− .014	ξ	
.718..	1.774	11.82	.317	− .010	ξ	
.785..	1.841	11.89	.279	− .021	ξ	
.857..	1.913	11.96	.282	+ .012	ξ	
.929..	1.985	12.03	.266	+ .005	π^5	
1168.692..	1.795	15.80	.323	+ .004	ξ	
.710..	1.813	15.82	.314	+ .003	ξ	
.765..	1.868	15.87	.282	− .006	ξ	
.830..	1.933	15.94	.257	− .008	ξ	
.909..	2.012	16.01	.283	+ .019	π^5	
1169.698..	2.801	16.80	.365	− .001	ξ	
.726..	2.829	16.83	.371	+ .006	ξ	
.797..	2.900	16.90	.347	− .015	ξ	Readings discordant
.830..	2.933	16.94	.375	+ .015	ξ	
.902..	3.005	17.01	.375	+ .020	π^5	
1170.691..	−0.159	17.80	.067	− .001	ξ	
.718..	−0.132	17.82	.000	− .013	ξ	
.776..	−0.074	17.88	(− .138)	(− .043)	ξ	Rejected
.801..	−0.049	17.91	− .127	+ .003	ξ	
.841..	−0.009	17.95	− .177	− .015	ξ, 2	
.865..	+0.015	17.97	− .151	+ .009	π^5	
.885..	0.035	17.99	− .192	− .048	π^5, 2	Readings discordant, half-weight
.909..	0.059	18.02	− .112	+ .006	π^5, 2	
1172.692..	1.842	19.80	+ .276	− .023	ξ	
.710..	1.860	19.82	.285	− .006	ξ	
.753..	1.905	19.86	.245	− .028	ξ	
.772..	1.922	19.88	.273	+ .005	ξ	
.794..	1.944	19.90	.270	+ .006	ξ	
.813..	1.963	19.92	.271	+ .009	ξ	
.867..	2.017	19.97	(.286)	(+ .021)	π^5	Probably smoke
.885..	2.035	19.99	(.300)	(+ .030)	π^5	
1178.781..	0.026	25.89	− .154	− .002	ξ	
.800..	0.045	25.91	− .128	+ .006	ξ	
.850..	0.095	25.96	− .063	− .006	π^5	
.882..	0.127	25.99	+ .022	+ .019	π^5	
.908..	0.153	26.01	.041	− .016	π^5, 2	
1179.719..	0.964	26.82	.371	+ .016	ξ	
.742..	0.987	26.85	.372	+ .015	ξ	
1182.663..	−0.045	29.77	(− .112)	+ .022	ξ	
.687..	−0.021	29.79	(− .134)	+ .022	ξ	
.708..	0.000	29.81	(− .165)	− .002	ξ	
.739..	+0.031	29.84	(− .130)	+ .018	ξ	Suspected influence of third body
.758..	0.050	29.86	(− .111)	+ .018	ξ	
.782..	0.074	29.89	(− .073)	+ .022	ξ	
.842..	0.134	29.95	(+ .001)	− .016	π^5	
.874..	0.166	29.98	(+0.093)	+0.011	π^5	

TABLE II—*Continued*

Date, G.M.T.	$3^d.95$ Phase	$34^d.46$ Phase	Difference of Magnitude	Residual	Comparison Star	Remarks
2421183.731..	1d.023	30d.84	o.M363	+o.M003	ξ	No effect from third body
.747..	1.039	30.85	.359	− .002	ξ	
1185.765..	3.057	32.87	.339	− .009	ξ	
.783..	3.075	32.89	.348	+ .002	ξ	
.848..	3.140	32.95	.337	− .001	π^5	
.868..	3.160	32.97	.325	− .011	π^5	
1186.632..	−0.029	33.74	− .141	+ .009	ξ	
.648..	−0.013	33.75	− .172	− .012	ξ, 2	
.683..	+0.022	33.79	− .158	− .003	ξ, 2	
.699..	0.038	33.80	− .157	− .016	ξ	
.733..	0.072	33.84	− .086	+ .012	ξ, 2	
.755..	0.094	33.86	− .057	+ .002	ξ	
.770..	0.109	33.88	− .035	− .004	ξ	
.783..	0.122	33.89	+ .014	+ .020	ξ, 2	
.842..	0.181	33.95	.095	− .016	π^5	
.857..	0.196	33.96	.143	+ .004	π^5, 2	
.870..	0.209	33.98	.160	.000	π^5, 2	
1187.622..	0.961	0.13	.354	− .001	ξ	
.641..	0.980	0.15	.354	− .002	ξ	
.711..	1.050	0.22	.353	− .009	ξ	
.739..	1.078	0.24	.354	− .010	ξ	
.840..	1.179	0.35	.374	+ .006	π^5	
1192.644..	2.030	5.15	.258	− .010	ξ	
.666..	2.052	5.17	.248	− .027	ξ	
.696..	2.082	5.20	.285	− .002	ξ	
.714..	2.100	5.22	.296	+ .002	ξ	
.741..	2.127	5.25	.310	+ .005	ξ	
.759..	2.145	5.26	.305	− .008	ξ	
1194.608..	0.041	7.11	− .127	+ .011	ξ	
.628..	0.061	7.13	− .112	+ .003	ξ	
.647..	0.080	7.15	− .077	+ .007	ξ	
.744..	0.177	7.25	+ .110	+ .007	ξ	
.762..	0.195	7.27	.146	+ .009	ξ	
.807..	0.240	7.31	.209	.000	π^5	
.831..	0.264	7.34	.225	− .014	π^5	
1197.655..	3.088	10.16	.351	+ .007	ξ	
.671..	3.104	10.18	.355	+ .012	ξ	
1202.743..	0.269	15.25	.240	− .005	π^4	
.759..	0.285	15.26	.229	− .033	π^4, 2	
.777..	0.303	15.28	.260	− .012	π^4, 2	
.817..	0.345	15.32	.283	+ .005	π^5, 2	
1203.620..	1.146	16.13	.375	+ .008	ξ	
.682..	1.208	16.19	.367	− .001	ξ	
.744..	1.270	16.25	.358	− .010	π^4	
1207.695..	1.268	20.20	.363	− .005	ξ	
1211.701..	1.321	24.21	.375	+ .007	ξ, 2	
1213.622..	3.242	26.13	.326	.000	ξ	
.645..	3.265	26.15	.347	+ .024	ξ	Smoke?
1222.575..	0.337	0.48	.261	− .016	ξ	
.606..	0.368	0.51	.290	+ .009	ξ	
1230.555..	0.410	8.46	.287	+ .002	ξ	
.636..	0.491	8.54	0.287	−0.010	ξ, 2	

TABLE II—*Continued*

Date, G.M.T.	$3^d.95$ Phase	$34^d.46$ Phase	Difference of Magnitude	Residual	Comparison Star	Remarks
2421232.575..	$2^d.430$	$10^d.48$	$0^M.373$	$+0^M.010$	ξ	
1234.560..	0.462	12.46	(.325)	+ .033	ξ	Third body?
1237.531..	3.433	15.44	.303	+ .002	ξ	
1247.512..	1.554	25.42	.355	− .007	ξ	
.536	1.578	25.44	.351	− .010	ξ	
.558..	1.600	25.46	.358	− .003	ξ	
.660..	1.702	25.56	.346	− .005	π^4, 2	
.676..	1.718	25.58	.322	− .024	π^4	
1252.522..	2.610	30.43	(.441)	+ .074	ξ	Third body?
.542..	2.630	30.45	(.429)	+ .061	ξ	
1253.668..	−0.197	31.57	.138	− .003	π^4, 2	
1256.577..	2.712	34.48	(.353)	(− .015)	ξ	Through smoke, rough test only
.600..	2.735	34.50	(.359)	(− .008)	π^4	
.639..	2.774	34.54	(.328)	(− .039)	π^4	
.654..	2.789	34.56	(.353)	(− .013)	π^4	
1258.508..	0.690	1.81	.306	− .019	ξ	
1259.511..	1.693	2.81	.344	− .010	ξ	
.562..	1.744	2.86	.330	− .007	ξ	
.617..	1.799	2.92	.318	+ .002	π^4	
.632..	1.814	2.93	.286	− .024	π^4, 2	
1265.537..	−0.188	8.84	.152	+ .028	ξ, 1	
.563..	−0.162	8.86	.088	+ .014	ξ, 1	
1269.547..	−0.131	12.85	(− .107)	− .118	ξ, 2	Third body?
.609..	−0.069	12.92	(− .168)	− .054	π^4	
1281.537..	−0.001	24.84	− .161	+ .002	π^4	
.581..	+0.043	24.88	− .138	− .002	π^4	
1296.541..	3.143	5.24	.301	− .037	π^4	Poor sky
1307.547..	2.289	16.24	.342	− .015	π^5	
1321.562..	0.491	30.26	(0.323)	(+0.026)	π^5	Smoke, rough test only

TABLE II—*Continued*

SERIES II

Date, G.M.T.	$3^d.05$ Phase	$34^d.46$ Phase	Difference of Magnitude	Residual	Comparison Star	Remarks
2421482.856..	−0.279	18.56	0.251	−0.009	*e*	
.895..	−0.240	18.60	.226	+ .014	ξ	
.935..	−0.200	18.64	.144	− .003	ξ	
1483.835..	0.700	19.54	.350	+ .014	*e*	
.872..	0.737	19.57	.351	+ .009	ξ	
1484.807..	1.672	20.51	.365	− .002	*e*	
.872..	1.737	20.57	.363	+ .015	ξ	
.924..	1.789	20.63	.338	+ .009	*e*	
1485.849..	2.714	21.55	.400	+ .022	*e*	
.894..	2.759	21.60	.391	+ .014	ξ	
1486.803..	−0.285	22.50	.281	+ .015	*e*	
.860..	−0.228	22.56	.222	+ .029	*e*	
.881..	−0.207	22.58	.160	+ .001	ξ	
.918..	−0.170	22.62	.102	+ .012	*e*, 2	
.936..	−0.152	22.64	.056	+ .001	ξ, 2	
1487.864..	0.776	23.57	.347	.000	*e*, 4	Smoke
1488.808..	1.720	24.51	.352	− .001	*e*	
.860..	1.772	24.56	.343	+ .007	*e*	
.917..	1.829	24.62	.318	+ .006	ξ	
1489.858..	2.770	25.56	.368	− .009	*e*	
.900..	2.812	25.60	.364	− .012	ξ, 2	
1490.799..	−0.242	26.50	.212	− .002	*e*, 2	Poor sky
1493.842..	2.801	29.54	.377	+ .001	ξ	
.905..	2.864	29.61	.380	+ .006	*e*	
1494.797..	−0.196	30.50	(.163)	+ .023	*e*	
.816..	−0.177	30.52	(.127)	+ .023	*e*	
.836..	−0.157	30.54	(.101)	+ .036	*e*	
.862..	−0.131	30.56	(+ .006)	− .005	ξ	Third Body?
.880..	−0.113	30.58	(− .019)	+ .004	ξ	
.901..	−0.092	30.60	(− .071)	− .011	*e*	
.919..	−0.074	30.62	(− .110)	− .016	*e*	
1495.798..	0.805	31.50	.347	− .002	*e*	Poor sky
.818..	0.825	31.52	.353	+ .002	*e*	
1496.923..	1.930	32.63	.291	+ .016	*e*, 2	Poor sky
.937..	1.944	32.64	.246	− .027	*e*, 2	
1499.832..	0.886	0.94	.340	− .018	ξ	
1500.778..	1.832	1.88	.328	+ .017	*e*	
.847..	1.901	1.95	.290	+ .007	ξ	
.893..	1.947	2.00	.280	+ .007	*e*	
.937..	1.991	2.04	.263	− .009	*e*, 2	Readings discordant
1501.842..	2.896	2.94	.371	− .001	ξ	
.861..	2.915	2.96	.373	+ .002	ξ	
1502.787..	−0.112	3.89	− .032	− .007	ξ	
.815..	−0.084	3.92	− .072	+ .006	ξ	
.841..	−0.058	3.94	− .102	+ .013	ξ	Probably smoke on this date
.862..	−0.037	3.96	− .138	− .001	ξ	
.879..	−0.020	3.98	− .176	− .026	ξ	
.899..	0.000	4.00	− .174	− .021	ξ	
1506.860..	+0.009	7.96	− .135	+ .015	ξ	
.891..	0.040	7.99	− .133	− .009	ξ	
1507.835..	0.984	8.94	0.364	−0.003	ξ	

TABLE II—*Continued*

Date, G.M.T.	3^d95 Phase	34^d6 Phase	Difference of Magnitude	Residual	Comparison Star	Remarks
2421510.780..	−0.021	11.88	−0.132	+0.017	*e*	
.803..	+0.002	11.91	− .154	− .002	ξ	
.825..	0.024	11.93	− .137	+ .003	ξ	
.842..	0.041	11.95	− .111	+ .012	ξ	
.860..	0.059	11.96	− .110	− .009	ξ	
1514.755..	−0.002	15.86	− .132	+ .021	*e*	
.776..	+0.019	15.88	− .126	+ .018	*e*	
.800..	0.043	15.90	− .127	− .006	ξ	
.819..	0.062	15.92	− .094	+ .003	ξ	
.839..	0.082	15.94	− .067	− .006	ξ	
.866..	0.109	15.97	− .010	+ .001	*e*	
.889..	0.132	15.99	+ .023	− .011	*e*	
.916..	0.159	16.02	.098	+ .010	μ	
.942..	0.185	16.05	.138	.000	μ	
1517.783..	3.026	18.89	.353	− .008	ξ	
1522.810..	0.148	23.92	.052	− .015	ξ	
.883..	0.221	23.99	.222	+ .024	μ	
1529.706..	3.091	30.81	(.293)	− .060	*e*, 2	Third body?
.778..	3.163	30.88	(.327)	− .018	*e*, 2	
1534.799..	0.278	1.30	(.339)	(+ .070)	ξ	Rejected
.842..	0.321	1.35	.264	− .022	*e*	Readings discordant
.894..	0.373	1.40	.308	+ .016	μ	
1535.790..	1.269	2.29	.390	+ .012	ξ	
.816..	1.295	2.32	.378	.000	*e*	
.853..	1.332	2.35	.363	− .014	*e*	
.880..	1.359	2.38	.392	+ .015	μ	
.904..	1.383	2.41	.376	.000	μ	
1536.756..	2.235	3.26	.345	− .013	ξ	
.771..	2.250	3.28	.337	− .026	ξ	
.800..	2.279	3.30	(.317)	(− .050)	*e*	No apparent reason for discrepancy
.817..	2.296	3.32	(.317)	(− .051)	*e*	
.861..	2.340	3.37	.367	− .003	μ	
.881..	2.360	3.39	.345	− .026	μ	
.902..	2.381	3.41	.351	− .020	μ	
1537.728..	3.207	4.23	.343	+ .003	ξ	
.753..	3.232	4.26	.350	+ .013	*e*	
1538.731..	0.257	5.24	.247	.000	ξ	
.751..	0.277	5.26	.270	+ .002	*e*	
.774..	0.300	5.28	.288	+ .006	ξ	
.795..	0.321	5.30	.282	− .004	ξ	
.809..	0.335	5.31	.296	+ .009	*e*	
.836..	0.362	5.34	.262	− .028	*e*	
.862..	0.388	5.37	.292	− .001	μ	
.885..	0.411	5.39	.291	− .005	μ	
.911..	0.437	5.42	.299	− .001	μ	
1548.749..	2.370	15.26	.374	+ .003	ξ	
.774..	2.395	15.28	.368	− .004	*e*	
1549.705..	3.326	16.21	.318	− .007	*e*	Poor sky
.724..	3.345	16.23	.332	+ .009	ξ	
.887..	3.508	16.39	.304	+ .005.	μ	
1551.687..	1.356	18.19	.367	− .010	ξ	
1553.692..	3.361	20.20	0.328	+0.007	ξ	

TABLE II—*Continued*

Date, G.M.T.	$3^d.95$ Phase	$34^d.6$ Phase	Difference of Magnitude	Residual	Comparison Star	Remarks
2421553.716..	$3^d.385$	$20^d.22$	$0^M.337$	$+0^M.020$	e	
1573.628..	3.531	5.53	.272	− .024	ξ	Cell near glowing point
1574.612..	0.562	6.52	.333	+ .015	ξ, 2	
.637..	0.587	6.54	.354	+ .032	e	
1579.610..	1.607	11.52	.354	− .017	ξ	
.637..	1.634	11.54	.358	− .011	e	
1580.623..	2.620	12.53	.363	− .015	e	
1583.562..	1.606	15.47	.375	+ .004	e	
.597..	1.641	15.50	.349	− .020	ξ	
1588.665..	2.756	20.57	.365	− .012	ξ	
1592.603..	2.741	24.51	.380	+ .003	e	
1596.589..	2.773	28.49	.388	+ .012	ξ	
.607..	2.791	28.51	.395	+ .019	e	
1598.561..	0.792	30.46	.347	− .001	e	No effect from third body
.579..	0.810	30.48	.354	+ .004	ξ	
1601.594..	−0.128	33.50	− .010	− .016	ξ	
1603.549..	1.827	0.85	.307	− .006	ξ	
1612.567..	2.938	9.87	.390	+ .021	ξ	
1613.534..	−0.048	10.84	− .129	− .003	ξ	
.581..	−0.001	10.88	− .143	+ .010	e	Readings discordant
1622.693..	1.205	20.00	.398	+ .020	μ	
1626.535..	1.094	23.84	.390	+ .015	ξ	
1628.567..	3.125	25.87	.370	+ .021	e	
.687..	3.245	25.99	.365	+ .030	μ	
1631.677..	2.282	28.98	.362	− .005	μ	
.701..	2.306	29.00	.380	+ .012	μ	
1635.531..	2.183	32.83	.338	− .002	ξ	
.550..	2.202	32.85	.323	− .024	e	
.569..	2.221	32.87	.356	+ .002	e	
.591..	2.243	32.89	.356	− .004	μ	
1663.569..	2.547	26.27	.370	− .007	μ	
1669.576..	0.647	32.27	.345	+ .016	μ	
1670.569..	1.640	33.27	0.369	0.000	μ	

curve. When less than three sets of measures were averaged the number is indicated by the figure following the comparison star. Where both the difference of magnitude and corresponding residual are in parentheses it indicates that the observation was entirely rejected; where the difference of magnitude only is in parenthesis the observation was not used in determining the short-period light-curve, but was used in the test for the third body.

In Table III are shown the reductions of the observations with π^5 Orionis. The light-curve of π^5 was referred to π^4 Orionis, and the correction on account of the variation of π^5 is given in the fourth column. The adopted values in the last column are

the sums of the corresponding numbers in the third and fourth columns plus 0.120 magnitude, the difference between π^4 Orionis and ξ Tauri.

TABLE III

COMPARISONS OF λ TAURI WITH π^5 ORIONIS

Date, G.M.T.	π^5 Phase	Observed Difference of Magnitude	Reduction to π^4 Orionis	Adopted Referred to ξ Tauri
2421164.929....	$0^d.770$	$+0^M.138$	$+0^M.008$	$+0^M.266$
1168.909....	1.049	+ .154	+ .009	+ .283
1169.902....	2.042	+ .289	− .034	+ .375
1170.865....	3.005	− .274	+ .003	− .151
.885....	3.025	− .313	+ .001	− .192
.909....	3.049	− .232	.000	− .112
1172.867....	1.307	+ .174	− .008	+ .286
.885....	1.325	+ .190	− .010	+ .300
1178.850....	3.589	− .145	− .038	− .063
.882....	3.621	− .059	− .039	+ .022
.908....	3.647	− .040	− .039	+ .041
1182.842....	0.181	− .084	− .035	+ .001
.874....	0.213	+ .006	− .033	+ .093
1185.848....	3.187	+ .228	− .011	+ .337
.868....	3.207	+ .217	− .012	+ .325
1186.842....	0.480	− .012	− .013	+ .095
.857....	0.495	+ .035	− .012	+ .143
.870....	0.508	+ .051	− .011	+ .160
1187.840....	1.478	+ .276	− .022	+ .374
1194.807....	1.044	+ .080	+ .009	+ .209
.831....	1.068	+ .097	+ .008	+ .225
1202.817....	1.653	+ .197	− .034	+ .283
1307.547....	2.771	+0.211	+0.011	+0.342

In Table IV are given the normal magnitudes on the basis of the period of 3.952941 days, together with the residuals from the final light-curve. In forming these normals it was desirable, especially for Series I, not to mix up in the same normal observations with different comparison stars, one of which is known to be a variable. Each normal usually comprises six sets or more, but when fewer than five sets were averaged that fact is indicated by the figure which follows the comparison star. The systematic corrections, which were applied to Series II before forming the normals, will be discussed later.

The normal magnitudes of Table IV are shown graphically in Figure 1, where open circles represent the results of Series I, and barred circles those of Series II. The normals of the first series which were of low weight, and also those referred to π^4 or π^5 Orionis, are indicated by crosses.

From the residuals in Table II there follow for the probable errors of a single observation: Series I, $\pm 0^{M}008$; Series II, $\pm 0^{M}009$. In computing these probable errors all poor observations were given full weight, and the residuals include all unknown variations of the comparison stars and outstanding errors of the light-curve. Treating the residuals of Table IV likewise as all of equal weight, the probable error of one normal is $\pm 0^{M}006$ for each series.

TEST FOR THE THIRD BODY

In view of the discordances which observers have reported for λ Tauri, it was expected that the photo-electric measures would show either large perturbations of the eclipse times, or actual

TABLE IV

NORMAL MAGNITUDES

SERIES I

Phase	Difference of Magnitude	Residual	Comparison Stars	Phase	Difference of Magnitude	Residual	Comparison Stars
$-0^{d}197$	$+0^{M}138$	$-0^{M}003$	π^4, 3	$1^{d}064$	$+0^{M}354$	$-0^{M}009$	ξ
− .175	+ .120	+ .020	ξ, 2	1.177	.371	+ .003	ξ
− .159	+ .067	− .001	ξ, 3	1.224	.366	− .002	π^4, π^5
− .132	.000	− .013	ξ, 3	1.294	.369	+ .001	ξ
− .039	− .134	+ .006	ξ	1.577	.355	− .006	ξ
− .011	− .174	− .012	ξ, 4	1.710	.334	− .014	π^4, 4
− .001	− .161	+ .002	π^4, 3	1.718	.337	− .009	ξ
+ .022	− .165	− .009	π^5	1.763	.318	− .013	ξ
.024	− .156	− .002	ξ	1.804	.318	+ .004	ξ
.041	− .137	+ .001	ξ	1.805	.305	− .009	π^4
.051	− .125	+ .003	π^4, π^5	1.842	.278	− .021	ξ
.066	− .099	+ .009	ξ	1.864	.284	− .005	ξ
.087	− .067	+ .004	ξ	1.908	.264	− .008	ξ
.111	− .020	+ .007	π^5	1.928	.265	− .001	ξ
.114	− .015	+ .006	ξ	1.954	.270	+ .007	ξ
.167	+ .068	− .016	π^5	1.998	.274	+ .012	π^5
.186	.128	+ .008	ξ	2.041	.253	− .018	ξ
.202	.152	+ .003	π^5, 4	2.089	.289	− .001	ξ
.252	.217	− .008	π^5	2.136	.308	− .001	ξ
.275	.236	− .015	π^4	2.289	.342	− .015	π^5, 3
.318	.272	− .003	π^4, π^5, 4	2.430	.373	+ .010	ξ, 2
.352	.276	− .002	ξ	2.815	.368	+ .002	ξ
.459	.287	− .005	ξ	2.922	.366	+ .005	ξ
.768	.326	− .009	ξ	3.066	.344	− .003	ξ
.816	.337	− .003	ξ	3.078	.353	+ .007	π^5
.962	.360	+ .005	ξ	3.096	.353	+ .010	ξ
.982	.360	+ .003	ξ	3.254	.336	+ .011	ξ
1.031	+0.361	+0.001	ξ	3.433	+0.303	+0.002	ξ, 3

TABLE IV—*Continued*

SERIES II

Phase	Difference of Magnitude	Residual	Comparison Stars	Phase	Difference of Magnitude	Residual	Comparison Stars
−0.277	+0.256	+0.003	*e*	0.813	+0.340	+0.001	*e*, ξ
− .236	+ .211	+ .008	*e*, ξ	0.993	.355	− .002	ξ
− .213	+ .181	+ .014	*e*, ξ	1.280	.370	+ .002	*e*, ξ, μ
− .180	+ .113	+ .004	*e*, ξ	1.371	.362	− .005	ξ, μ
− .135	+ .013	− .006	ξ	1.621	.352	− .008	*e*, ξ
− .093	− .062	− .001	ξ	1.656	.348	− .010	*e*, ξ, μ
− .048	− .126	+ .005	ξ	1.748	.343	+ .007	*e*, ξ
− .024	− .145	+ .009	*e*, ξ	1.820	.311	+ .003	ξ
− .006	− .164	− .001	*e*, ξ	1.893	.293	+ .015	*e*, ξ
+ .005	− .168	− .005	*e*, ξ	1.962	.257	− .005	*e*
.011	− .154	+ .008	ξ	2.207	.329	− .008	*e*, ξ
.027	− .142	+ .009	*e*, ξ	2.244	.336	− .013	ξ, μ
.045	− .132	+ .002	ξ	2.271	.334	− .022	ξ, μ
.056	− .128	− .006	ξ	2.340	.344	− .016	μ
.077	− .090	− .001	ξ	2.387	.351	− .010	*e*, ξ, μ
.125	− .004	− .004	*e*	2.632	.364	− .004	*e*, μ
.159	+ .060	− .008	ξ, μ	2.757	.369	+ .002	*e*, ξ
.208	.160	+ .001	μ	2.783	.374	+ .008	*e*, ξ
.272	.248	.000	*e*, ξ	2.831	.364	− .001	*e*, ξ
.319	.268	− .007	*e*, ξ	2.921	.368	+ .007	ξ
.362	.275	− .004	*e*, μ	3.124	.345	+ .005	*e*, ξ
.417	.274	− .012	μ	3.273	.331	+ .009	*e*, μ
.604	.331	+ .018	*e*, ξ, μ	3.369	.322	+ .012	*e*, ξ
0.743	+0.339	+0.007	*e*, ξ	3.521	+0.277	−0.011	ξ, μ

occulting effects of the third body. During the first season, however, no discrepancy was noted until J.D. 2421252, January 23, 1917, when two observations of three sets each, with ξ Tauri as comparison star, made λ Tauri unusually bright, the residuals later being determined as 0.074 and 0.061. ξ Tauri was promptly compared with π^4 Orionis, and this measure made ξ Tauri only 0.014 fainter than usual. There was nothing apparent in the observing conditions which could account for this sudden brightening up of λ Tauri, and the discordance was some seven or eight times the probable error of an observation.

The next large discordance was on J.D. 2421269, February 9, 1917, when two comparison stars were used. The mean of two sets with ξ Tauri made λ Tauri faint by 0.118, while three sets with π^4 Orionis gave a residual of 0.054 in the same direction. As these measures were taken during the primary eclipse of the close pair, a careful search has been made for an error in the

recorded times or in the reductions, but the results are apparently correct.

At the end of that season when all of the results were arranged on the 34.6-day phase, it was seen that the discrepancies had occurred at two places in the cycle, separated by about half the period. These were at the approximate phases 13 and 30.5 days, zero phase being counted from the hypothetical time of the third

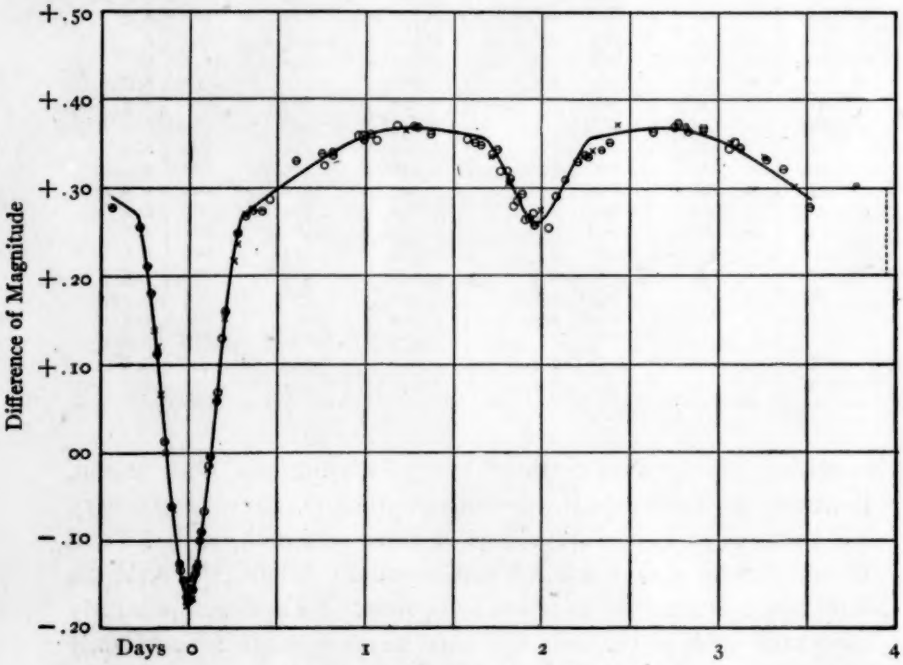

FIG. 1.—The light-curve of λ Tauri

body eclipsing the center of mass of the close system, as computed from the spectroscopic elements. Therefore, if we could assume that the third body was about four days ahead of its predicted place, we should have a clue to the cause. A triply eclipsing system would present interesting observational difficulties, as the eclipses of the close components by the distant member would each be displaced over a range of several days due to the motion about the center of mass. I have no idea, however, as to what

would cause the total light of the system to be increased by 6 or 7 per cent when the third body is nearly in front of the others, though at the present writing I have suspicions of a similar brightening up at the beginning and end of the eclipses of another star. The observations of λ Tauri were carried through the second season with the special object of detecting the effect of the third body, but also to polish up the light-curve for the close pair.

The residuals of Table II have been arranged according to the phase of the longer period, combining observations only on the same night, and the result of this test is shown in Figure 2. As before, open circles indicate the observations of Series I, barred circles those of Series II, while crosses indicate the means of fewer

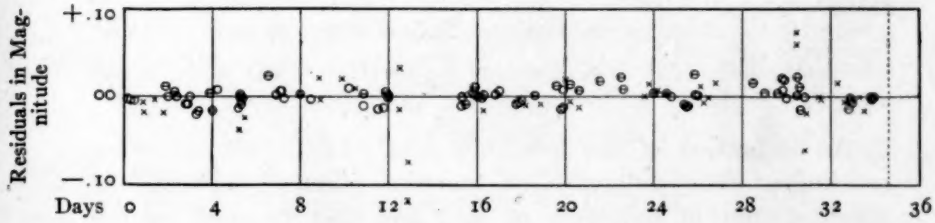

FIG. 2.—Test for the third body of λ Tauri; period = 34.6 days

than five sets for either series. There are apparently four nights involved in the larger discrepancies, two when the system was faint and two when it was bright, also one or two other nights near the same phase when the light was changing abnormally. On the other hand there are about an equal number of observations in the vicinity of phase 30–31 days where nothing unusual was noted.

My conclusion is that at best we have here only a suspicion of an effect due to a third body, which in any event must be small, and it is not worth while to continue the search further. After spending two seasons on the attempt I am willing to let someone else try his hand at detecting discrepancies of less than a tenth of a magnitude, which come at intervals of seventeen days, and which if they exist at all must be subject to large irregularities.

COMPARISON WITH THE SPECTROSCOPIC ELEMENTS

A further test of the anomalies in this system is given by a comparison of the photometric results with the spectroscopic orbit, since on the simple eclipse theory the epoch of primary minimum should be when the bright body is 90° from the node.

TABLE V

TIMES OF MINIMA

Source	Elements	Computed Minimum	Difference
		J.D.	
Spectroscopic orbit.....	.....	2417945.240	.....
v. Aretin.....	J.D. 2399607.527+3^d.952941·E	7945.220	−0^d.020
Photometric 1916, Series I.....	2421194.573+3^d.952941·E	7945.255	+ .015
Photometric 1917, Series II.....	2421506.850+3^d.952941·E	7945.250	+ .010
Photometric 1917, with corrected period.....	2421506.850+(3.952952·E)	7945.240	(0.000)

An inspection of the available data leads to the following conclusion. The difference of fourteen minutes between the observed time of minimum in 1917 and that required by the spectroscopic elements from plates taken in 1907–1914, corresponds to an error of 1 km/sec. in the velocity-curve, of 0^M.02 in the light-curve, or of one second in the adopted period. It will therefore require more evidence than we have at present to establish any real discordance between the photometric and spectroscopic results.

THEORY OF THE SYSTEM OF λ TAURI

In deriving the elements of the system of λ Tauri the method and notation of Russell have been used. The first step was to derive the law of variation between eclipses. Using the normals farther than 0.35 day from the times of primary and secondary minima, there was found:

$$\left.\begin{aligned} \text{Series I, Magnitude} &= 0^M.3576 - 0^M.0474 \cos\theta - 0^M.0542 \cos^2\theta \\ &\quad \pm 13 \qquad \pm 25 \qquad\quad \pm 47 \\ \text{Series II, Magnitude} &= 0^M.3740 - 0^M.0420 \cos\theta - 0^M.0710 \cos^2\theta \\ &\quad \pm 17 \qquad \pm 20 \qquad\quad \pm 41 \end{aligned}\right\} \quad (1)$$

where θ is the phase angle, and the magnitude is referred to ξ Tauri. The difference between the first terms could be due to a progressive variation of the total light of the system or to a change in the comparison star from one year to the next. The coefficients of $\cos\theta$ agree within half a hundredth of a magnitude. The difference between the coefficients of $\cos^2\theta$ could be interpreted to mean a change in the ellipticity of figure of the components, but the discrepancy of 30 per cent is too large to be considered in this connection. By inspection of the graphs of both series it was seen that by a shift of a hundredth of a magnitude the curve between minima for Series II would fit the observations of Series I very well; and it was independently found that during the primary eclipse a systematic difference of a hundredth of a magnitude in the same direction also existed. It was therefore thought best to apply the correction of $0^M.010$ to all magnitudes of Series II. It was also found graphically that the time of primary minimum for Series II was 0.005 day early on the basis of the times computed from the first season. The two series were treated therefore as follows:

Series I.—Original observations in Table II and all normals were unchanged.

Series II.—Original phases and magnitudes in Table II were unchanged, but corrections of +0.005 day to the phases and $-0^M.010$ to the observed differences of magnitude were applied before forming the normals in Table IV. The residuals for Series II, given in Table II, were derived after the corrections had been applied.

The normal magnitudes for the two seasons were thus made homogeneous, and the next step was to rectify the magnitudes during the eclipses, which was accomplished from (1) by adding $0^M.0542\cos^2\theta$ to the observed magnitudes, then reducing to light and adding $0.0418(1+\cos\theta)$ in light units. At primary minimum the normals before and after zero phase were combined on one branch of the curve. The observations near secondary minimum were used only to determine the minimum light at that phase, as the form of the curve, with less than a tenth of a magnitude range, was of no service in deriving the elements.

The spectroscopic eccentricity of 0.060 gives 2.010 days for the phase of the secondary minimum, but as the photometric results are not sufficient to establish an accurate time in this part of the curve it has seemed sufficient to neglect the eccentricity and assume the central phase of the secondary to be at the phase equal to one-half of the period, or 1.976 days.

TABLE VI

COMBINED NORMALS, PRIMARY MINIMUM

Phase	Observed Magnitude	Rectified Light	Residual Light	Series
$0^d.006$	$-0^M.166$	0.705	−0.002	II
0.008	− .170	.703	− .006	I
0.021	− .147	.716	+ .006	II
0.023	− .159	.709	− .003	I
0.040	− .138	.721	+ .001	I
0.046	− .129	.726	+ .003	II
0.057	− .113	.735	+ .006	I
0.066	− .109	.738	− .001	II
0.095	− .051	.773	+ .005	I
0.109	− .033	.785	− .002	II
0.121	− .009	.799	− .001	I
0.147	+ .036	.828	− .007	II
0.167	+ .085	.862	+ .001	I
0.193	+ .136	.899	+ .003	I
0.194	+ .136	.899	+ .001	II
0.224	+ .196	.943	+ .011	II
0.264	+ .226	.964	− .013	I
0.274	+0.252	0.984	+0.002	II

TABLE VII

RECTIFIED CURVE

	Magnitude	Range	Light	Loss of Light
Maximum	$0^M.405$	.....	1.000	.....
Primary minimum	+0.029	$0^M.376$	0.707	0.293
Secondary minimum	0.315	0.090	0.920	0.080

The combined normals for primary minimum, using both branches of the curve, are given in Table VI. After some trials the constants of the curve were adopted as in Table VII. These constants with the combined rectified normals are the basis of the elements.

It is obvious at the outset that λ Tauri furnishes a case where the elements will be poorly determined, first because of the small

range of about $0^M.4$ in a partial eclipse, and second because of the undetermined light of the third body. In allowing for the effect of the ellipsoidal figure of the bodies, we have from the last term of (1) 0.0542 in magnitude = 0.0487 in light, and hence

$$z = 2 \times 0.0487 = 0.097.$$

But this value of z is too small if there is light from the third body, which would reduce the apparent variation of the eclipsing pair from whatever cause. In view of this uncertainty, and for convenience, all computations have been on the basis:

$$z = 0.10.$$

In Russell's method for a partial eclipse, we have to find the function $\chi\ (k, a_0, \frac{1}{4})$ which is derived from the form of the curve at primary minimum. If we consider the total light at any time as coming from a pair of overlapping disks plus the light of the third body, the effect would be the same if we should extinguish the third member and give its light to the uneclipsed member of the close pair. The χ function is therefore independent of the light of the third body.

In using the secondary minimum to determine the elements we have for two bodies

$$a_0(L_1 + L_2) = 1 - \lambda_1 + \frac{1 - \lambda_2}{k^2},$$

but with the third body, if $L_1 + L_2 + L_3 = 1$, there follows

$$a_0 = \frac{1}{1 - L_3}\left(1 - \lambda_1 + \frac{1 - \lambda_2}{k^2}\right).$$

We now determine a_0 and k by the intersection of the curve

$$\chi\ (k, a_0, \tfrac{1}{4}) = \text{constant},$$

and the system of curves

$$a_0 = \frac{1}{1 - L_3}\left(0.080 + \frac{0.293}{k^2}\right)$$

which may be drawn for different assumed values of L_3. As there is no way of determining L_3, the elements must remain uncertain within certain limits, between zero and a quantity such that L_2 and L_3 together shall not be large enough for their combined spectra to show on the plates. This limit of L_2+L_3 is assumed to be one-third of L_1, or

$$0<L_3<\frac{L_1}{3}-L_2.$$

When the first approximate solution was made, it was found that L_3 must be as large as 0.13 for the curves giving a_0 and k to intersect. Thus with only two eclipsing bodies a solution was impossible, but it was possible on the assumption of a third body giving 0.13 of the total light. This was a striking case of the "astronomy of the invisible," but a critical examination shows that the material is not sufficient to establish the light effect of the third body in this way. It was found that any value of the χ function between 1.76 and 1.84 would satisfy the observations about equally well. Even if χ could be determined there is a large range of the values for each element of the system, as is shown in Table VIII, which is computed for $C=0.125$, $D=0.0695$, $C/D=\chi=1.80$.

TABLE VIII

ELEMENTS FOR χ $(a_0, k, \frac{1}{4})=1.80$

k	0.641	0.625	0.65	0.70	0.75	0.80	0.90	1.00
a_0	1.00	.95	.84	.73	.655	.60	.52	.45
L_1	.71	.79	.83	.82	.796	.76	.70	.65
L_2	.08	.09	.10	.11	.122	.13	.15	.18
L_3	.21	.12	.07	.07	.082	.11	.15	.17
$L_1/(L_2+L_3)$	2.4	3.8	4.9	4.6	3.9	3.2	2.3	1.9

From Table VIII it is seen that k may range from 0.6 to 0.8, but that for $k=0.9$, L_1 would be only 2.3 times (L_2+L_3), and the additional spectra should be detected. Similar tables were made for $\chi=1.76$ and 1.84. For $\chi=1.76$ there is a solution for $k=0.6$, and no solutions for larger values of k, except that there is just a possibility for $k=1.0$. For $\chi=1.84$ the least

value of L_3 would be 0.20, and any solution would give too much light for the two fainter bodies. The possibility of the large body being in front at primary minimum seems to be entirely excluded, as this would also give too great a value for L_2+L_3. Taking the various combinations, the plausible range of the different quantities is

$$\begin{aligned} k &= 0.6 - 1.0 \\ a_0 &= 0.4 - 1.0 \\ L_1 &= 0.75 - 0.9 \\ L_2 &= 0.1 - 0.2 \\ L_3 &= 0.0 - 0.15 \\ L_1/(L_2+L_3) &= 3 \quad -10. \end{aligned}$$

The value $k=0.75$ is therefore probably correct within 20 per cent, and on this basis, with $\chi=1.80$, the remaining elements will give a fair idea of the nature of the system. Using Russell's formulae there are found for the radii of the bodies and the inclination:

$$\begin{aligned} a_1 &= 0.299 \\ ka_1 &= 0.224 \\ \cos i &= 0.202 \end{aligned}$$

From the ellipsoidal constant $z=0.10$ we have, following Russell, for the axes of the ellipsoids

$$a:b:c = 1.00:0.95:0.92.$$

The effect of the polar flattening is too small to consider as changing the computed inclination.

The final light-curve during eclipse was computed both by Russell's method, using the constants a_0, k, C, and D, and also independently from the elements by an adaptation of Harting's method, using Blazko's table.[1] The necessary changes because of elliptical disks were of course taken into account.

The graph in Figure 1 shows a satisfactory agreement of the curve with the observations, the only outstanding discordance being near the beginning and end of the secondary minimum, where a lowering of the curve by about a hundredth of a magnitude

[1] *Annales de l'observatoire astronomique de Moscou*, 5, 104, 1911.

would make a better agreement. This error is largely due to the adoption of the curve between minima as wholly from Series I, but in view of the systematic difference between measures of the two seasons, and the fundamental uncertainty in the final elements, it was not considered worth while to go through the work again. Likewise there appears no advantage in a case like this in making a solution for disks darkened at the limb.

TABLE IX

LIGHT-CURVE OF λ TAURI

Phase	Difference of Magnitude	Phase	Difference of Magnitude
$0^d.00$	$-0^M.163$	$1^d.80$	$0^M.316$
±0.05	− .129	1.85	.295
±0.10	− .048	1.90	.275
±0.15	+ .051	1.95	.263
±0.20	.146	1.976	.261
±0.25	.222	2.00	.262
±0.30	.271	2.05	.274
±0.307	.273	2.10	.294
+0.40	.284	2.15	.315
0.50	.298	2.30	.334
0.75	.333	2.25	.351
1.00	.358	2.282	.357
1.25	.368	2.50	.365
1.273	.368	2.680	.368
1.50	.363	2.75	.367
1.669	.357	3.00	.355
1.70	.352	3.25	.326
1.75	0.335	3.50	0.291

In his discussion of the system of λ Tauri, Schlesinger has shown that the relative distances of the second and third bodies from the brightest one can be closely estimated, and on the assumption that the primary has 2.5 times the mass of the eclipsing satellite he finds the distances of the three bodies from the centroid of m_1 and m_2 to be respectively 3,200,000 km, 8,000,000 km, and 50,000,000 km. He used sin i=0.96, which is near enough the present value, 0.98. On this basis and the sun's radius of 695,500 km, there follow when the sun's mass, radius, and density are each taken as unity

m_1=2.5	a_1=4.8	ρ_1=0.025
m_2=1.0	a_2=3.6	ρ_2=0.024
m_3=0.4		ρ_0=0.025

The mean density of the close pair, ρ_0, follows from the values of the period and of a_1 and k, and it is only by a coincidence that the independent assumption of the relative masses gives nearly equal densities for the two components. Lacking eclipse measurements of the third body, we have no clue as to its diameter and density. In Figure 3 the close system is drawn to scale, and in spite of the uncertainties this probably gives a rough idea of the conditions and dimensions of the two stars.

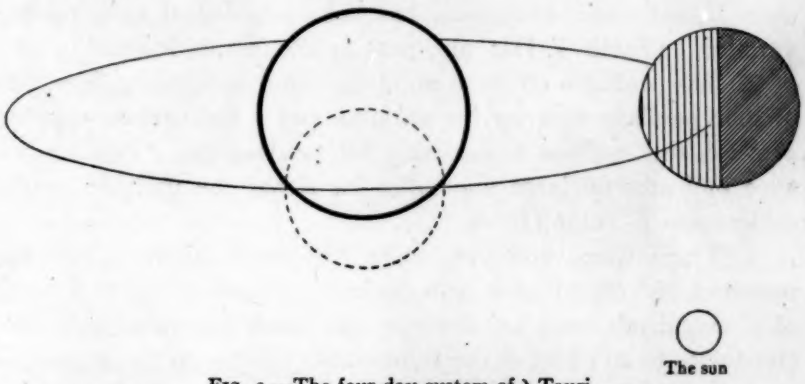

FIG. 3.—The four-day system of λ Tauri

The second body in the system of λ Tauri is similar to the companion of Algol in being brighter on the side toward the primary, in fact this radiation effect is apparently present in all close systems where the bodies are of unequal brightness. Using the formula[1] for the value of the albedo, μ, on the basis of Lambert's law of reflection there follows

$$\mu = 2.7.$$

The corresponding albedo with the reflection law of Seeliger has been computed with an unpublished formula which I have derived or:

$$\mu = 3.8.$$

The conclusion from these figures, which is also plausible from other considerations, is simply that since theoretically the albedo cannot exceed unity the extra light from one side of the second body must be due in large part to the heating effect of the intense radiation from the primary.

[1] *Astrophysical Journal*, 33, 397, 1911.

SUMMARY

We may now bring together the principal results of this paper, emphasizing some points which have not been brought out before.

1. A thorough study of λ Tauri has been made with the photo-electric photometer, and it is found that this star is a normal eclipsing system, with two minima and continuous variation between eclipses. (Fig. 1, Table IX.)

2. Of the five comparison stars used, all of which are of spectral class B, only one, π^5 Orionis, has been established as a regular variable. (Tables I, III; also next article in this *Journal*.)

3. The probable errors of an observation comprising three sets of four readings each on the variable and a comparison star are $\pm 0^{M}008$ and $\pm 0^{M}009$ respectively for two seasons. This accordance indicates no large anomalies for either the variable or the other stars. (Table II.)

4. There were, however, some half-dozen nights when the measures of λ Tauri were systematically discordant up to a tenth of a magnitude, and an attempt was made to trace these discrepancies to an effect of the third body. Although the anomalies occur at two places in the 34.6-day cycle which are separated by half the period, we have at best only a suspicion that the third body is the cause. (Fig. 2.)

5. A comparison of the four-day light-curve with the Allegheny spectroscopic elements shows a good agreement of the eclipse times within the errors of the observed data. (Table V.)

6. There is a systematic difference of about a hundredth of a magnitude between the light of λ Tauri as measured in 1916–1917 and in 1917–1918. This may be ascribed to a change in the total light of the variable, in the light of the comparison star ξ Tauri, or to an unsuspected instrumental change; but there is not enough evidence at hand to make a decision on this matter. (Table II, Series I and II, also [1] Series I and II, also (1).)

7. Subject to the unavoidable uncertainties and with the assumptions which have been made, the plausible elements for the triple system of λ Tauri for uniform disks are as follows.

It is a pleasure to make acknowledgment of the work of various persons whose collaboration has made possible the results in this paper. The development of the photo-electric cells has been due

to Dr. Jakob Kunz, of our department of physics of the University. It happened that the particular quartz cell described was made with Dr. Kunz's apparatus during his absence by Mr. L. A. Welo, with some assistance from myself. I am indebted to Dr. Elmer Dershem for various improvements in the photometer, and for

TABLE X

SUMMARY OF ELEMENTS OF λ TAURI

		Adopted	Range
From the Light-Curve			
Ratio of radii of first and second bodies	k	0.75	0.6 −1.0
Area of second body obscured at minimum	a_0	0.655	0.4 −1.0
Light of first body	L_1	0.796	0.75−0.9
Light of bright side of second body	L_2	0.122	0.1 −0.2
Light of faint side of second body	L_2-2b	0.038	0.0 −0.12
Light of third body	L_3	0.082	0.0 −0.15
Ratio of surface brightness of bright sides of first and second bodies	J_1/J_2	3.7	.....
Major radius of first body	a_1	0.299	.....
Major radius of second body	a_2	0.224	.....
Cosine of inclination	$\cos i$	0.202	.....
Ellipsoidal constant	z	0.10	.....
Ratio of axes of ellipsoids	b/a	0.95	.....
	c/a	0.92	.....
Mean density of close pair, sun = 1	ρ_0	0.025	.....
Computed albedo of second body, Lambert's law	μ	2.7	.....
Computed albedo of second body, Seeliger's law	μ	3.8	.....
From Allegheny Spectroscopic Elements Assumed, $m_1 = 2.5\ m_2$		Sun = 1	
Mass of first body	m_1	2.5	.....
Mass of second body	m_2	1.0	.....
Mass of third body	m_3	0.4	.....
Major radius of first body	a_1	4.8	.....
Major radius of second body	a_2	3.6	.....
Density of first body	ρ_1	0.025	.....
Density of second body	ρ_2	0.024	.....

about half of the observations in 1917–1918; also to various students, in particular to Mr. P. H. Lucas, for assistance in observing, to Miss Iva Hamlin for reductions, and to Mr. L. L. Steimley for check computations.

This work is a portion of that accomplished under two grants of three hundred dollars each from the Draper fund of the National Academy of Sciences.

UNIVERSITY OF ILLINOIS OBSERVATORY
February 1920

THE ELLIPSOIDAL VARIABLE STAR, π^5 ORIONIS

By JOEL STEBBINS

ABSTRACT

Ellipsoidal variable star, π^5 Orionis.—The *light-curve* for this variable has been determined with reference to π^4 Orionis by a series of observations made with a photo-electric photometer in 1916–1918. The results, corrected for atmospheric extinction, show that this star is a continuous variable with a double period equal to the time of revolution in the spectroscopic orbit, 3.70 days (Fig. 1). No eclipses are apparent. The evidence indicates that the variation is due entirely to the ellipsoidal shape of the components, resulting from their mutual attraction. The elongation of the brighter component, computed from the amplitude of light variation, must be something more than 5 per cent. This is the first case where ellipsoidal variation unaccompanied by eclipses has been detected by the author.

Spectroscopic binary, π^4 Orionis.—The photo-electric photometric observations indicate that the *light variation* of this binary is little or none.

During the winter of 1916–1917, in the first series of measures of λ Tauri with the photo-electric photometer, it was found that one of the comparison stars, π^5 Orionis, was giving discordant results, and it was natural to suspect that this star was itself variable.[1] At the same time π^5 Orionis was being used as a standard in light-tests of the spectroscopic binary π^4 Orionis.[2] Through an oversight the reference to the spectroscopic orbit of π^5 Orionis had been missed, and it was not until the light variation was well established that Lee's orbit[3] was found. As some of the comparisons of λ Tauri with π^5 Orionis had been made during primary minima of λ Tauri when no other comparison star was available, it was decided to determine the light-curve of π^5 with π^4 Orionis as standard, and then on the basis of this curve to use π^5 for λ Tauri. In the sequel it turned out that π^4 has little or no light variation, but even if it had the problem of determining both light-curves by comparison of two spectroscopic binaries of known periods could easily be solved by successive approximations.

[1] *Publications of the American Astronomical Society*, **3**, 272, 1916.

[2] Baker, *Publications of the Allegheny Observatory*, **1**, 110, 1908.

[3] *Astrophysical Journal*, **38**, 180, 1913.

In Table I are the observations. The first date is October 31, 1916, and it will be noted that sixteen observations were made in 1916–1917, and nine in 1917–1918. Each difference of magnitude is the mean of three sets of four readings on each star, except where a different number is noted in the remarks. The differences are in the sense indicated, a plus sign meaning that π^4 was brighter than π^5. The correction for atmospheric extinction in the third column was derived by multiplying the tabular visual extinction by the factor in the fourth column. This factor was determined

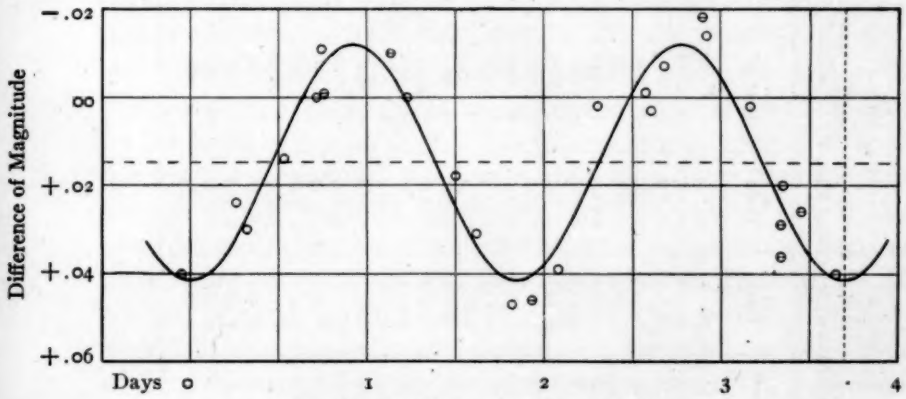

FIG. 1.—The light-curve of π^5 Orionis

as a matter of routine for each night, the adopted value being found from measures of stars of known relative brightness; from the absolute light effects from standard stars, the instrumental conditions being constant over long intervals; and from the observer's estimate of the quality of the sky. The adopted factor was more or less arbitrary but was formed without reference to its effect upon the accordance of the results.

The phases for π^4 and π^5 are counted from the hypothetical eclipse times computed from the respective orbits.

$$\pi^4, \text{ minimum} = \text{J.D. } 2418287.584 + 9^d.5191 \cdot E$$
$$\pi^5, \text{ minimum} = \text{J.D. } 2417922.565 + 3^d.70045 \cdot E$$

In Figure 1 the differences of magnitude are plotted according to the phase of π^5, and it is seen that this star is a continuous

TABLE I

OBSERVATIONS OF π^4 AND π^5 ORIONIS

Date, G.M.T.	$\pi^5-\pi^4$, Observed	Extinction	Factor	π^4 Phase	π^5 Phase	$\pi^5-\pi^4$, Corrected	Residual	Remarks
J.D. 2421159.932	+0M018	−0M020	1.7	7d099	3d174	−0M002	−0M011	4 sets
1186.889	+ .046	− .032	2.0	5.499	0.527	+ .014	+ .005	2 sets, poor
1197.728	+ .048	− .024	2.0	6.818	0.265	+ .024	− .007	
.783	+ .050	− .020	2.0	6.873	0.320	+ .030	+ .003	
1199.769	+ .019	− .017	1.7	8.859	2.306	+ .002	− .013	5 sets
1202.787	+ .060	− .029	2.6	2.358	1.623	+ .031	− .003	
1203.743	+ .017	− .018	1.8	3.314	2.579	− .001	+ .005	
.774	+ .021	− .018	1.8	3.345	2.610	+ .003	+ .011	
1207.782	+ .010	− .024	2.2	7.353	2.918	− .014	− .005	
1230.713	+ .058	− .018	1.6	1.727	3.646	+ .040	− .001	
1247.646	+ .057	− .018	1.8	9.141	2.077	+ .039	+ .005	
1253.691	+ .025	− .025	1.8	5.667	0.721	.000	+ .006	
.708	+ .018	− .029	1.8	5.684	0.738	− .011	− .004	
1269.592	+ .065	− .018	1.8	7.434	1.820	+ .047	+ .006	
1281.556	+ .011	− .018	1.8	4.975	2.683	− .007	+ .004	
1302.578	+ .060	− .042	2.5	6.958	1.502	+ .018	− .007	
1550.769	+ .032	− .033	3.0	6.653	0.763	− .001	+ .007	
1554.833	+ .019	− .029	2.6	1.198	1.127	− .010	− .004	
1596.748	+ .065	− .029	2.2	6.036	3.337	+ .036	+ .013	
1603.710	+ .006	− .024	2.1	3.479	2.898	− .018	− .008	6 sets, discordant
1611.670	+ .046	− .020	1.8	1.920	3.457	+ .026	− .007	
1622.653	+ .053	− .024	2.2	3.384	3.338	+ .029	+ .006	
.669	+ .042	− .022	1.8	3.400	3.354	+ .020	− .005	
1628.647	+ .065	− .019	1.6	9.378	1.932	+ .046	+ .006	6 sets, discordant
1631.649	+0.039	−0.039	3.0	2.861	1.233	0.000	−0.001	

variable with double period equal to the time of revolution in the spectroscopic orbit. There are apparently no eclipses, and since the two minima are approximately equal and the spectrum is single, the variation is due entirely to the ellipsoidal shape of the components. The light variation is expressed by

$$\text{Difference of magnitude} = +0^{M}.0146 + 0^{M}.0267 \cos 2\theta,$$
$$\pm 10 \qquad \pm 14$$

where θ is the phase angle, and the coefficients and their probable errors were derived by a least-squares solution from the twenty-five observations. The probable error of one observation of Table I is $\pm 0^{M}.0048$. Since the mean difference between π^4 and π^5 has been measured as $0^{M}.0146$ with a probable error of a thousandth of a magnitude, it is worth while to examine the measures and see if this apparent accuracy is not illusory. A test of the accordance is given by averaging the results for the two seasons separately, which gives $0^{M}.0142$ and $0^{M}.0154$, a discrepancy of $0^{M}.0012$, which is small enough to indicate that neither π^4 nor π^5 is changing progressively at a rapid rate. However, an inspection of the extinction correction in Table I shows that with corrections ranging from $0^{M}.017$ to $0^{M}.042$ the agreement between the two seasons must be in part accidental. The mean values of the extinction for the two seasons are $0^{M}.022$ and $0^{M}.027$, and it could easily happen that there is a systematic error of several thousandths in either season. These two stars do not give a favorable case for the elimination of the extinction, as π^4 is always higher in the sky than π^5, but if we take two nearly equal stars of the same spectral type, close together and near the zenith, the mean of twenty-five observations with the photo-electric photometer ought to have a probable error as small as $0^{M}.001$, assuming, of course, that the stars are not variable.

The coefficient of 2θ gives readily a measure of the least ellipticity of figure of the components of π^5 Orionis.

$$2 \times 0^{M}.0267 = 0^{M}.0534 = 0.048 \text{ in light,}$$
$$\text{Ratio of axes, } b/a = 0.952.$$

We conclude that the two bodies are elongated by mutual attraction by something more than 5 per cent, though of course this measure probably refers only to the brighter component. This is the first case in the tests of spectroscopic binaries where I have detected the ellipsoidal variation without there also being eclipses, and there must of course be many other such objects awaiting discovery.

The results in this paper were secured with the same assistance and support that were acknowledged in the preceding paper on λ Tauri.

UNIVERSITY OF ILLINOIS OBSERVATORY
February 1920

A MODIFICATION OF THE ELECTRON THEORY OF DISPERSION TO ACCOUNT FOR THE CHANGE OF THE REFRACTIVE INDEX WITH TEMPERATURE

By E. O. HULBURT

ABSTRACT

Dispersion formula involving temperature, according to a modified electron theory.—Various observers have found that the decrease of refraction index with increasing temperature is approximately linear. To explain this fact, the author has modified the Lorentz dispersion formula by introducing the *assumption* that, for the range of spectrum involved, the variation with temperature is determined by electrons of a single type, upon which the medium acts only with a *restoring force which increases linearly with the absolute temperature*, as is to be expected from the law of equipartition of energy. The new formula has four arbitrary constants (equation 6). The *agreement with observed results* for a number of transparent liquids is within 0.2 per cent for a considerable range of wave-length and temperature and clearly indicates that the above assumption is a step in the right direction, though it is too simple to correspond accurately with the facts.

1. *Introductory.*—Measurements of the refractive indices of liquids at various temperatures and for various wave-lengths of monochromatic radiation are recorded in the Landolt-Börnstein physical tables. More recently K. G. Falk[1] has made a systematic study of the refractive index of fourteen organic liquids for four wave-lengths of light in the visible region of the spectrum for temperatures between 15° and 75° C. Most of the observers have concluded that within the error of experiment the refractive index increased linearly with decrease in temperature. Forch and Kucera[2] have given the results of their determinations of the refractive indices of six liquids at temperatures below 0° C. for sodium light in the form of empirical equations which, in addition to the first power, contained higher powers of the temperature. The numerical coefficients of the higher powers were, however, small, showing that the deviations from the linear relation were slight.

No previous attempt has been made to explain quantitatively these facts on the basis of any of the theories of dispersion.

[1] *Jour. Amer. Chem. Soc.*, **31**, 86 and 806, 1909.

[2] *Physikalische Zeitschrift*, **3**, 132, 1901–1902.

In the present paper an assumption is made concerning the mechanism of the dispersing medium which leads to a modification of the dispersion formula, enabling the refractive index at various temperatures to be computed.

2. *Theoretical.*—In the following discussion we shall consider only isotropic, transparent media, for the experimental data at hand preclude consideration of any but the simplest cases. It is assumed that the only force due to the medium which acts on the dispersion electron is of an elastic nature. Therefore there is no frictional force (i.e., absorption) and no external magnetic field. We use the electron theory of dispersion as given by H. A. Lorentz,[1] and write the equation of motion of the dispersion electron of a single type

$$m\frac{d^2\xi}{dt^2}=\sqrt{4\pi}\, e\,(E_x+\sigma\sqrt{4\pi}\, e\, N\xi)-f\xi. \qquad (1)$$

ξ and E_x are the X-components of the displacement of the electron from its equilibrium position and the electric force on the electron, respectively. There are similar equations for the Y- and Z-components. f is a certain positive constant which depends upon the structure and properties of the particle or molecule in which the electron is situated, such that $-f\xi$ is the X-component of the elastic force or the force of restitution. The charge on the electron in c.g.s. electrostatic units is e, its mass is m. N is the number of such electrons per unit volume. σ is a constant which Lorentz has shown to be approximately $1/3$ for isotropic media.

Let ϵ be the base of natural logarithms, and let all dependent variables of (1) contain the time only in the factor $\epsilon^{i\frac{2\pi c}{\lambda}t}$, where $\frac{2\pi c}{\lambda}$ is the frequency, λ the wave-length of the vibration, and c the velocity of light *in vacuo*. The solution of (1) gives the refractive index μ as determined by the relation

$$\frac{1}{\sigma+\frac{1}{\mu^2-1}}=\frac{4\pi N\frac{e^2}{m}}{\frac{f}{m}-\frac{4\pi^2c^2}{\lambda^2}}. \qquad (2)$$

[1] *Theory of Electrons*, 1909, p. 132.

This equation expresses μ in terms of the constants of a single type of dispersion electron. There may be other types of dispersion electrons in the medium with constants peculiar to the type, so that in the more general case the right-hand member of (2) becomes a summation of similar terms, one term for each type. For this case the complete dispersion formula is

$$\frac{1}{\sigma+\frac{1}{\mu^2-1}}=\sum\frac{C_s}{\frac{f}{m}-\frac{4\pi^2c^2}{\lambda^2}}, \tag{3}$$

where $C_s=4\pi N_s\frac{e^2}{m}$, and the subscript s denotes the sth type of electron.

We assume we are dealing with a region of the spectrum in which the change of refractive index with wave-length is determined by the electrons of a single type, so that in the summation of (3) all the terms except one may be replaced by a quantity q_1 which is independent of λ. Then (3) becomes

$$\frac{1}{\sigma+\frac{1}{\mu^2-1}}=q_1+\frac{C_1}{\frac{f}{m}-\frac{4\pi^2c^2}{\lambda^2}}. \tag{4}$$

We now consider the effect of temperature-changes upon μ as expressed by (4). It is assumed that σ and q_1 remain constant throughout any temperature-change. C_1 is a function of the temperature, but as it is proportional to the number of electrons per unit volume, and hence to the density, its changes with temperature may be calculated from a knowledge of the density of the substance at various temperatures.

There remains for consideration the quantity f, which is the elastic force of restitution acting on the electron. *f is assumed to increase linearly with the absolute temperature.* This assumption may be interpreted to result logically from the law of the equipartition of energy. This law states that during the temperature-change of a substance the energy associated with each degree of freedom within the substance varies proportionally with the

absolute temperature. When the law of equipartition is applied to a substance composed of molecules which have no internal modes of motion, it has been shown that the kinetic energy of the molecule is proportional to the absolute temperature. If the molecule possesses internal modes of motion, as will be the case if it contains movable electrons, the same law of equipartition of energy may be assumed to dictate the distribution of energy among these modes, and it follows in a particular case that the force of restitution of the electron is a positive linear function of the absolute temperature. We then write

$$f=f_0\,(1+\beta\theta), \tag{5}$$

where θ is the temperature Kelvin, f_0 is the value of f at the zero of temperature, and β is a positive constant.

Introducing (5) into (4), the dispersion formula becomes finally

$$\frac{1}{\sigma+\frac{1}{\mu^2-1}}=q_1+\frac{C_1}{\frac{f_0}{m}(1+\beta\theta)-\frac{4\pi^2c^2}{\lambda^2}}. \tag{6}$$

3. *Application to observation.*—Since formula (6) contains no term denoting absorption, its application is limited to substances which are transparent to the wave-lengths in the region of the spectrum concerned. Furthermore, the refractive index of the substance at constant temperature must conform to the relation of refractive index and wave-length given by equation (4).

The application of the dispersion formula (6) to several substances gave the same general results. The case of benzaldehyde is discussed in detail below. The method of computation and the results for this substance are typical of those for all the substances examined.

The refractive index of benzaldehyde for wave-lengths 434, 486, 589.3, and 656 $\mu\mu$, from the observations of Falk[1] for a range of temperature between 20° and 70° C., are shown by the straight lines of Figure 1. The density of the substance was also measured throughout this range of temperature. The determination of μ was

[1] *Loc. cit.*

accurate to about 0.0001, and Falk considered that a straight line represented the variation of μ with θ, when λ is constant, within the error of experiment.

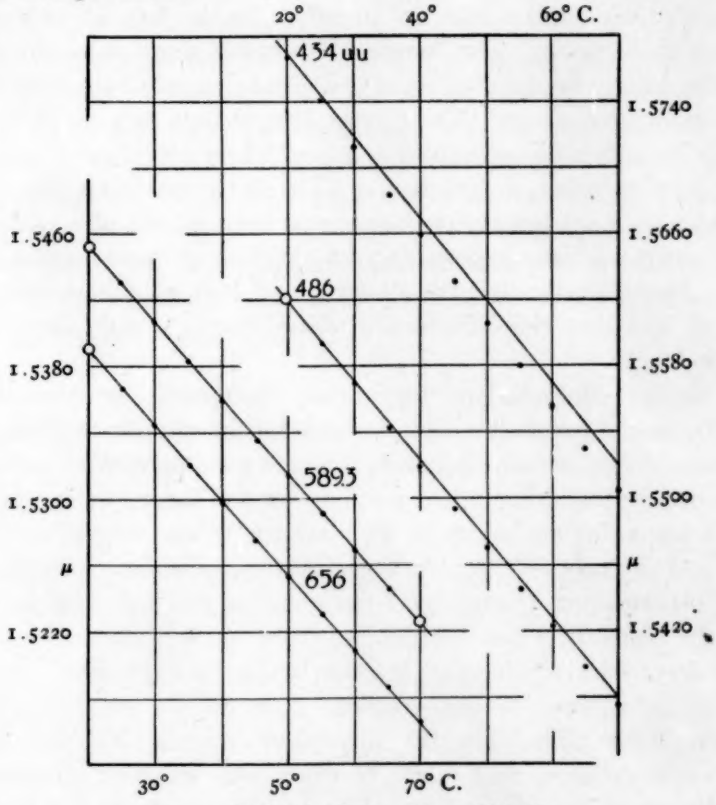

FIG. 1.—Refractive index of benzaldehyde

Introducing the values of the wave-length and of the corresponding refractive index from Figure 1 at 20° C., namely,

λ	486$\mu\mu$	589.3	656
μ	1.5621	1.5453	1.5391

and at 70° C., namely, $\lambda = 589.3$ $\mu\mu$ and $\mu = 1.5225$, into the dispersion formula (6) to determine the four constants, we find

$$q_1 = 0.50887 \qquad C_1 = 40.931 \times 10^{30} \text{ at } 20° \text{ C.}$$

$$\beta = 7.8778 \times 10^{-4} \qquad \frac{f_0}{m} = 83.882 \times 10^{30}$$

In the computations it is to be remembered that σ is $\frac{1}{3}$, and that for any temperature C_1 is proportional to the density. With the above values for the constants of (6) μ has been computed for various temperatures from 20° to 70° C. for the four wave-lengths given in Figure 1. The computed values of μ are shown by dots in the figure; the four values of μ used to determine the constants are shown by circles. The temperature-change in μ as given by (6) is linear to a first approximation, and it is seen that the computed μ-θ lines have but slight curvature for all the wave-lengths. For the longer wave-lengths the agreement between the observed and computed μ-θ lines is good, but for the shorter wave-lengths it is seen from Figure 1 that the observed and computed lines differ in slope, and that the difference in slope increases with decreasing wave-length.

Similar computations for carbon disulphide, acetylacetone, and α-monobromnaphthalene yielded results similar to those for benzaldehyde, namely, equation (6) gave good agreement between the observed and computed μ-θ lines for the longer wave-lengths, with increasing deviations as the wave-length was decreased.

4. *Discussion.*—The accuracy with which equation (6) expresses the observations, though none too great, is still sufficient to lead to the belief that the assumption given by (5), which possesses certain physical significance, is a step in the right direction. Manifestly substances do not conform very exactly to the several assumptions underlying the dispersion formula (6), and only approximate agreement might be expected. Without attempting to discuss all the assumptions, there are two which deserve further mention. In the first place, the assumption that the absorption is zero, and hence that the change of absorption with temperature has a negligible effect upon the refractive index, is questionable. In the second place, changes in q_1 due to temperature have been assumed to produce no noticeable changes in μ. Such an assumption is also hardly permissible. Undoubtedly q_1 does change with the temperature, since it is the contribution to the dispersion equation (6) from the other types of electrons, and in view of the fact that in all the cases examined q_1 was nearly as large as one-half of the refractive index its temperature variations probably affect

μ appreciably. However, the quantitative data on the absorption of the substances and on the refractive index and its temperature-changes are hardly numerous enough to justify more extended numerical computations.

5. *Summary.*—In order to explain the observed temperature variations of the refractive index of transparent isotropic substances, the assumption has been made that the force of restitution acting on the dispersion electron increases linearly with the absolute temperature. This assumption when introduced into the dispersion equation deduced from the electron theory of Lorentz resulted in an equation for the refractive index which was found to express the results of observation with a certain degree of exactness.

JOHNS HOPKINS UNIVERSITY
December 1919

FINE STRUCTURE OF THE NEAR INFRA-RED ABSORPTION BANDS OF THE HALOGEN ACIDS

By WALTER F. COLBY

ABSTRACT

Absorption spectra of diatomic gases; a quantum theory of the fine structure of the infra-red bands.—The postulate of Bjerrum, that the fine structure of these bands is due to the rotation of the molecules in stationary states corresponding to the quantum relation, has proved very fruitful. The purpose of the present paper is to test the hypothesis by comparing the *theoretical results for the simplest molecular model of the Bohr type* with the experimental results recently obtained by Imes for the *halogen acids*. For HF, HCl, and HBr, the observed interatomic vibration frequencies come out respectively 18, 40, and 47 per cent less than the theoretical, while the computed separations of the pairs of nuclei are respectively 38, 54, and 59 per cent less than the values given by Imes. The variation with mass is explained as a screening effect which has been neglected in the elementary theory. The agreement between theory and experiment is sufficient to lend support to the Bjerrum hypothesis, and suggests the importance of developing a more adequate theory involving fewer simplifying assumptions.

Recent experimental work in the fine structure of absorption spectra in the near infra-red has been greatly stimulated by the very fruitful postulate of Bjerrum[1] concerning the mechanism of this absorption. He explains the structure of the bands in this region as due to the vibration of the atoms in the molecule modified by the rotation of the molecule as a whole about an axis normal to the line of vibration. Following the requirements of the quantum theory, the molecule is assumed to be capable of stationary states of rotation defined by the equation

$$\omega = nh/2\pi I .$$

n indicates successive integers. The rotational frequencies are thus

$$\nu_r = nh/4\pi^2 I .$$

Calling the interatomic vibration frequency ν_0, the foregoing theory states that absorption will occur at frequencies given by the equation

$$\nu = \nu_0 \pm \nu_r .$$

[1] *Nernst Festschrift*, p. 90, 1912.

Assuming that ν_0 is not modified by the rotational velocity and consequently not a function of n, this equation would require a series of absorption lines symmetrically grouped about ν_0. The series would have two regions of maximum intensity symmetrically placed, corresponding to the most probable value of ω. A small variation of ν_0 with ω would cause a crowding of the bands on one side and a separation on the other. Considerable experimental evidence has been obtained to verify the original hypothesis in these details. It is apparently applicable likewise in the study of band spectra, a success which will greatly strengthen its present position.

The first experimental work which presented these absorption regions with sufficient dispersion to show the fine structure was done by Eva von Bahr in 1913.[1] She used a prism spectrometer and was able to distinguish and measure a considerable number of lines in the bands of a group of diatomic and triatomic gases. The separation of the lines in the simpler spectra gave rotation frequencies which it was possible to identify with absorption lines in the far infra-red, and moments of inertia computed from these and the quantum relation were of the order of magnitude demanded by the kinetic-gas theory. With low dispersion these regions appear as doublets. With these and also with the fine structure slightly resolved, von Bahr was able to measure relative intensities and find that the rotation velocities approximated a Maxwellian distribution. When the temperature was raised the doublet separated and its members became flatter and broader. The maximum shifted outward according to the requirements of the Maxwell law for change of most probable velocity with temperature. The recent work, especially that of Sleator[2] and Imes[3] in this laboratory and of Kemble and Brinsmade[4] at Harvard, has extended the dispersion and resolution of the fine structure very materially. Work now in progress confirms qualitatively the earlier statement that the rotational velocities conform quite closely to a Maxwellian distribution, even broken up as they are into the discrete

[1] *Verhandlungen der deutschen Physikalischen Gesellsohaft*, **15**, 710, 1913.

[2] *Astrophysical Journal*, **48**, 125, 1918. [3] *Ibid.*, **50**, 251, 1919.

[4] *Proceedings of the National Academy of Sciences*, **3**, 420, 1917.

regions of preferred velocities. The great contribution of the new work, however, has been in pushing the resolution to its apparent limit and in determining so precisely the frequencies of these ultimate members. The effect of rise of temperature of the absorbing gas now appears very definitely not to displace these individual lines but simply to sweep the position of maximum intensity outward toward higher values of n. Present work is also using this means of assembling data on the higher members.

The work of Imes has presented the absorption of three of the halogen acids with remarkable precision. It is the object of the present paper to make use of this data in checking the order of magnitude of the variation of ν_0 with n as shown by a molecular model of the Bohr type and to see whether it is possible to express ν_0 for such a model by as strikingly simple a function of n as Imes has found it to be. It will be possible also to check the variation of I with n.

The simplest model of the Bohr type is that first proposed by him,[1] which has the great advantage of being electrically symmetrical. HCl has been chosen as a type, since the results are most complete for that gas. The chlorine atom is assumed to contribute one electron only to the valence ring. The hydrogen atom also contributes one. The H nucleus and the Cl nucleus with remaining electrons are then treated as point charges, an assumption which may depart considerably from the details of the model, but which is sufficiently near for a first treatment. The nuclei thus each possess one positive charge and differ only in mass. Bohr has written the equilibrium conditions for such a system and for this problem it is only necessary to add one term to take care of the molecular rotation. This term is in fact of the same type as the one Bohr has used to indicate the centrifugal force on the electrons in the valence ring. $2x$ indicates the distance between the nuclei, y the radius of the valence ring, M is the reduced mass of the rotating system $[I = M(2x)^2]$, other symbols have their usual significance. The equations are

$$n^2h^2/4\pi^2\,(2x)^3M + e^2/(2x)^2 = 2e^2x/(x^2+y^2)^{3/2}$$
$$h^2/4\pi^2y^3m + e^2/(2y)^2 = 2e^2y/(x^2+y^2)^{3/2}.$$

[1] *Phil. Mag.*, **26**, 857, 1913.

The first term in each equation gives the centrifugal force and recognizes the fact that the angular momentum $=nh/2\pi$. For the electrons, n is set equal to unity. The influence of the molecular rotation on the electrons is neglected. A slight computation shows that the electrons have no appreciable effect on the reduced mass and also that the centrifugal force on the electrons is of a much lower order of magnitude than the electric forces on them parallel to the molecular axis. The equations are easier to discuss if one replaces y by a new variable α by means of the substitution

$$y = x \tan \alpha .$$

It is possible then to solve for x and n^2 in terms of α. For $n^2=0$, $\alpha=60°$. α decreases slightly as n^2 grows and becomes $59°27'$ for $n^2=104$. Although it has not been possible to form a closed expression for x in terms of n^2, a curve of corresponding values shows the relation to be very nearly linear for $n^2<100$. One may express these quantities as a series in ascending powers of the very small quantity $(60°-\alpha)$. These series are rapidly convergent and easily obtained. They contribute nothing new to the present discussion but are convenient tools for future work. The following values of the constants were used.

$$\begin{aligned} M &= 1.613 \ 10^{-24} \\ h &= 6.55 \ 10^{-27} \\ e &= 4.77 \ 10^{-10} \\ m &= 9.01 \ 10^{-28} \end{aligned}$$

The linearity between x and n^2 is then given by

$$x = 2.917 \ 10^{-9} + 1.23 \ 10^{-12} n^2 .$$

Imes gives at $n=0$, $x=6.4 \ 10^{-9}$. This agreement is not close, but one must note that Imes obtains in the case of HF $x=4.7 \ 10^{-9}$ and for HBr, $x=7.1 \ 10^{-9}$, and that our equations do not involve M when $n=0$. The observed variation must therefore be due to the screening effect of the inner electrons in the halogen atom. In this calculation the foregoing model should consequently be more nearly checked by HF, which we find to be the case. The centroid of this group of molecules lies always very near the halogen nucleus. We should expect, therefore, that many variations with atomic

number must be explained rather by this screening effect than by difference of mass. A model which is otherwise satisfactory but assumes as we have done that the halogen atom may be replaced electrically by a point charge will yield limiting values which the experimental data will approach as the atomic number decreases. As soon as this material is extended and completed the dependence of these frequencies on atomic number can be more satisfactorily studied.

Imes has found an apparent linearity between x^2 and n. It is very difficult to establish a relation here since the variation is so extremely small. He finds that the variation of x from the first to the tenth member is about $50 \cdot 10^{-12}$. Our formula gives $123 \cdot 10^{-12}$.

To investigate the vibration frequency we shall assume that the amplitudes involved in the absorption in this region are small enough so that we may set the restoring force proportional to the displacement. ν_0 is then given by $\frac{1}{2\pi}\sqrt{F/M}$, where

$$F = d/d(2x)(n^2h^2/32\pi^2Mx^3 + e^2/4x^2 - 2e^2\cos^3 a/x^3).$$

A differentiation of the second of the equilibrium equations allows an evaluation of da/dx. Although the calculation is fairly laborious, it is straightforward. One can obtain an expression of ν_0 in terms of a, and with the help of the previous relation between a and n^2 a graphical solution is possible for ν_0 in terms of n^2. Here again the curve is surprisingly closely approximated by a straight line, the equation of which may be written

$$\nu_0 = 1.4485\ 10^{14} - 4.63\ 10^{10}\ n^2.$$

Imes's equation reads

$$\nu_0 = 0.8661\ 10^{14} - 0.9\ 10^{10}\ n^2.$$

We must note again that the observed variation of the constants of this equation with atomic number of the halogen is much too great to be due to difference of mass alone.

For HF $\nu_0 = 1.1887\ 10^{14} - 6.20\ 10^{10} n^2$.
HCl $\nu_0 = 0.8661\ 10^{14} - 0.90\ 10^{10} n^2$.
HBr $\nu_0 = 0.7677\ 10^{14} - 0.69\ 10^{10} n^2$.

The assumed model may again be thought of as an idealization of the halogen acid molecule, which one may expect the actual cases to approach as the atomic number decreases. The discussion of the influence of atomic number will be resumed here also as more measurements are made.

To show that the centers of these line pairs are subject to a variation of the same magnitude as the supposed molecular model would suffer with rotation is striking evidence of the authenticity of the assumption that the fine structure of these bands is due to this rotation. Work is now satisfactorily in progress to extend the number of observed lines of the HCl absorption region at 3.4 μ by raising the temperature of the absorbing gas. It is hoped thus to test these variations with much more material. The second absorption region at slightly less than double this frequency is also being extended and we may hope it to yield more information as to the possibility of a second stationary amplitude of vibration.

A second treatment will endeavor to remove at least some of the foregoing simplifying assumptions. This would consist in using the more satisfactory Kossel model, the at least partial recognition of the space-configuration of the halogen atom, and, most important, the scrutiny of the validity of assuming simple harmonic motion. In respect to this last point, if one assumes an equation of motion involving the square of the displacement the Sommerfeld phase integral still allows a definite statement of the problem of stationary states. By inspecting the standard solution of this new equation of motion one may foresee that the foregoing results will be changed in the right direction. The more complicated solution requires some ingenuity of treatment but may prove valuable in accounting for the higher frequency regions which apparently share the same structure.

PHYSICAL LABORATORY, UNIVERSITY OF MICHIGAN
January 1920

NOTE ON THE AIR LINES IN SPARK SPECTRA FROM λ 5927 TO λ 8719[1]

By PAUL W. MERRILL

ABSTRACT

Air lines in condensed spark spectra, λ 5927 to λ 8719.—As an essential preliminary to the investigation of spark spectra in this region, a study of the air lines has been begun by comparing spectra obtained with different elements as electrodes, both in air and in oxygen. (1) The *wave-lengths* of 58 air lines, so determined, are given in Table I with an accuracy which is limited by the fact that the lines are usually ill-defined and sometimes broad and hazy. Twenty of them correspond to known H, N, O, or A lines, including six lines of the *red spectrum of argon* and four *oxygen lines* which had not previously been observed in spark spectra. Except for two more, which are probably due to oxygen, the rest have not, as yet, been identified. (2) All of the identified lines are found to be *shifted with reference to vacuum tube spectra;* the increase of wave-length is about 0.1 A for the *oxygen lines* and two *nitrogen lines*, 0.3 A for three other N lines, 0.6 A for the *argon lines*, and 0.7 A for *hydrogen alpha* and for three other N lines. (3) The *effect of adding self-inductance* is to weaken the two *nitrogen lines*, λλ 6482 and 6610.

Spectrum of condensed spark in oxygen.—Several identifications of oxygen and nitrogen lines by previous observers using vacuum tubes are confirmed and the doubtful identification of λ 7157 as due to oxygen is strengthened.

In the spectrum of the condensed spark in air there appear numerous lines which are to be ascribed to the gases of the surrounding atmosphere rather than to the material of the electrodes. These lines may be stronger and more numerous than the electrode lines. They are usually ill-defined and often broad and nebulous. Lines of nitrogen, oxygen, hydrogen, and argon are present in the spectrum of the disruptive discharge in ordinary air.

Very little information in regard to spark spectra beyond λ 6600 is available at present, although the need for it is becoming pressing. The recognition of the air lines is an essential preliminary to the investigation of spark spectra in a new region; and since it is always of interest to study spectral lines as produced under different modes of excitation, the observation of the air lines in spark spectra is desirable for the additional reason that many of

[1] *Contributions from the Mount Wilson Observatory*, No. 183.

these lines occur both in vacuum tube discharges and in the tube-arc.[1] Moreover, certain of these lines are present in astronomical spectra. The best-known examples are the hydrogen lines, but the presence of the spark lines of oxygen and nitrogen in stellar spectra has been recognized for some time.

In the present investigation fifty-eight lines which are thought to be due to air have been observed in spark spectra from λ 5927 to λ 8719. These lines have all been photographed in the spectrum of the spark between electrodes of at least two elements, carbon and aluminum. Most of them have been photographed with several other elements also. For the region of wave-length greater than λ 7000 the air lines are the most striking features of the spectra of all the elements observed, including Al, C, Ca, Co, Cu, Fe, Ni, Ti, so that the photographed infra-red spark spectra of these elements appear much alike.

Wave-length measurements.—The lines which have been measured in the present investigation appear in the first column of Table I. Their relative intensities are in the second column. All the lines are more or less hazy and diffuse; those which are especially so are marked "h." The voltage employed was 8000, the capacity about 0.1 microfarad. No auxiliary spark-gap was used. No special temperature control was provided for the electrodes but care was taken that they did not become excessively hot, for in that case the characteristic spark features are weakened and the spectrum tends to become like that of the arc.

At λλ 6456 and 6950 are broad hazy patches which cannot be accurately measured. λ 7479 is a curious feature extending from λ 7476.7 to λ 7482.1. It is markedly unsymmetrical, being stronger toward the violet. Presumably it is composed of several ill-defined lines completely blended. Most of the other lines are fairly symmetrical, though lack of symmetry was especially noted for λλ 7424, 7442, and 7468. With high dispersion the lines of the oxygen triplet λλ 7772, 7774, 7775 appear slightly unsymmetrical.

All the lines have been measured with a dispersion of 16.8 A per mm; the stronger ones with 3.7 A per mm, and λλ 6482 and

[1] Arthur S. King and Paul W. Merrill, "Recent Observations of Tube-Arc Spectra Especially in the Infra-Red," *Physical Review*, 14, 271, 1919.

TABLE I

Air Lines in Spark Spectra

I. A.	Int.	Probable Error	Previous Observers	Probable Error	Vacuum Tubes
5927.83	5	.02	5927.88	.03	5927.57 N
5931.78	7	.02	5931.85	.04	5931.50 N
5941.54	11	.02	5941.65	.05	5941.49 N
5952.33	4	.02	5952.48	.03	5952.20 N
6160.72	1h	(.04)			
6171.0	3h	(.4)	6170.4	.1	
6284.22	1	(.03)	6284.4	(.2)	6283.29 N
6358.13	0h	(.08)			6356.78 N
6370.92	0h	(.06)			
6379.52	3h	.03	6379.7	(.2)	6378.79 N
6456	vh				O
6482.054	7	.004	6482.045	.01	6481.34 N
6563.5	12vh	.06	6563.3	.1	6562.79 H
6610.39	8	.007	6610.41	.04	6610.06 N
6640.7	0	(.3)			
6654.78	2	.05			
6721.25	1	.02			
6811.9	0	(.3)			
6864	00				
6887.61	1	(.06)			
6950	vh				
6965.95	1	.04			6965.43 A
7067.6	0	(.08)			7067.22 A
7157.36	9	.01			7157. O
7384.53	1	(.04)			7383.98 A
7424.04	8	.02			
7432.9	0	(.1)			
7442.70	10	.01			
7458.7	0	(.1)			
7468.72	14	.02			(7474. O)?
7479					
7504.84	0	(.04)			7503.87 A
7515.16	0	(.07)			7514.65 A
7635.70	1	(.07)			7635.11 A
7772.07	10	.01			7771.97 O
7774.33	7	.01			(7774.00)O
7775.60	6	.01			(7775.67)O
7947.83	4	.01			
7951.10	3	(.06)			
7952.3	2	(.2)			
8185.26	4	(.02)			
8188.42	4	(.02)			
8200.72	1	(.03)			
8211.12	2	(.02)			
8216.72	7	(.02)			
8223.48	4	(.02)			
8230.20	0	(.05)			
8242.80	4	(.03)			
8446.84	5	(.03)			8446.42 O
8594	00				
8630.02	0	(.04)			

TABLE I—*Continued*

I. A.	Int.	Probable Error	Previous Observers	Probable Error	Vacuum Tubes
8680.63	2	(.04)			
8683.70	1	(.04)			
8686.41	0	(.04)			
8692	00				
8703.8	00	(.2)			
8712.00	0	(.1)			
8719.2	00	(.2)			

6610 with 1.8 A per mm in addition, first-order grating spectra being used in every observation. The spectra having 16.8 A per mm and some of those having 3.7 A per mm were obtained on films, the remainder on glass plates. The wave-lengths were determined in most cases by reference to iron-arc comparison spectra, usually in the second order, but on some plates in the first order. Wherever possible, however, lines in the spark spectra themselves were used as standards. For this purpose the exposures to the calcium and copper sparks were especially suitable. The wave-lengths of the metallic lines were taken from *Scientific Papers of the Bureau of Standards*, Nos. 312 and 324.

In the fourth column appear the mean wave-lengths found by previous observers.[1] In the third and fifth columns are the probable errors; those in parentheses, depending on a small number of determinations, have been estimated; the others have been computed from the residuals in the usual way.

Hydrogen.—The hydrogen line Hα which appears on all of the photographs is exceedingly diffuse and difficult to measure. Measurements on a considerable number of exposures, however, have given the wave-length λ 6563.5, with a probable error of 0.06 A. The effective wave-length thus found is 0.7 A greater

[1] The principal sources from which these wave-lengths have been taken are Otto Schulemann, "Das Funkenspektrum des Indiums," *Zeitschrift für wissenschaftliche Photographie*, **10**, 263, 1911–1912; Franz Joseph Kasper, "Messungen am Silberspektrum," *ibid.*, p. 53; A. Kretzer, "Untersuchungen über das Antimonspektrum," *ibid.*, **8**, 45, 1909–1910; Wilhelm Schwetz, "Die Spektren des Wismuts," *ibid.*, p. 301; Matthias Aretz, "Über den langwelligen Teil des Kupferfunken- und Kupferbogenspektrum," *ibid.*, **9**, 256, 1910–1911; J. M. Edet and E. Valenta, "Wellenlängenmessungen im roten Bezirke der Funkenspektra," *Sitzungsberichte Akad. der Wiss. Wien, Math-naturw. Kl.*, **118**, IIa, 511, 1909.

than that obtained from vacuum tubes.[1] The values of previous observers give a slightly smaller displacement, 0.5 A. The interpretation of this apparent displacement should be considered in connection with the similar displacements shown by the lines of nitrogen and argon.

Nitrogen and oxygen.—Most of the air lines are due to nitrogen and oxygen. The vacuum tube wave-lengths of nitrogen are by Porlezza,[2] those of oxygen are by Runge and Paschen,[3] reduced to the international system.[4]

Confirmation of several of the identifications was obtained by operating the spark in an atmosphere of oxygen. The oxygen used was a commercial product and probably was not free from nitrogen, as the nitrogen lines were not eliminated from the spectrum and in fact were not greatly reduced in intensity. When the exposures in oxygen were so timed as to give the same general intensity as exposures in air, only two lines were found to be certainly weakened, namely λλ 6482.054 and 6610.39. The following lines were strengthened in an atmosphere of oxygen: λλ 6158, 6456, 7003, 7157.36, 7772.07, 7774.33, 7775.60, 8446.84. The lines at λλ 6158, 6456, and 7003 are very wide and diffuse. The first and third are absent or very weak in air and do not appear in Table I. λλ 6158 and 6456 probably correspond to triplets, and λ 7003 to a single line, all observed by Runge and Paschen in the series spectrum of oxygen. The last four lines are well-known oxygen lines, and λ 7157.36 is probably due to oxygen. A line at approximately this wave-length was observed in an oxygen tube by Runge and Paschen, though not included by them in their table of oxygen lines.

The exposures with the spark in oxygen did not give sufficiently great changes in the relative intensities of the remaining lines in

[1] W. E. Curtis, "Wave-Lengths of Hydrogen Lines and Determination of the Series Constant," *Proceedings of the Royal Society*, A **90**, 605, 1914.

[2] C. Porlezza, "Lo Spettro a righe dell'azoto in tubo di Geissler," *Atti R. Accademia dei Lincei*, Serie 5, **20**, 584, 1911.

[3] Runge and Paschen, *Annalen der Physik*, **61**, 641, 1897; Paschen, *Annalen der Physik*, **27**, 562, 1908.

[4] K. W. Meissner, "Untersuchungen und Wellenlängenbestimmungen im roten und infraroten Spektralbezirk," *Annalen der Physik*, **50**, 713, 1916.

Table I to make it possible to assign their origins. They may be due to nitrogen, but this is not certain except for those of shorter wave-length corresponding to lines found by previous observers in nitrogen tubes. The identification of the remaining lines must await further work with the spark in atmospheres of pure gases or with vacuum tubes containing pure gases.

Comparison of the spark wave-lengths of nitrogen lines with those by Porlezza in the vacuum tube shows a large positive displacement. For eight lines the mean displacement is 0.43 ± 0.08 A, the probable error being computed as though the shift were actually of the same amount for all lines. This is not necessarily

TABLE II

WAVE-LENGTHS OF OXYGEN LINES

SPARK	TUBE RUNGE AND PASCHEN*	ARC		SUN MEISSNER*
		Meissner*	Meggers and Kiess†	
7772.07.....	7771.97	7771.98	7771.93	7771.97
7774.33.....	7774.00	7774.19	7774.14	7774.20
7775.60.....	7775.67	7775.42	7775.43	7775.42
8446.84.....	8446.42	{8446.36 8446.76}	8446.42	{8446.38 8446.78

* K. W. Meissner, *Annalen der Physik*, 50, 713, 1916.
† *Bulletin of the Bureau of Standards*, 14, 637, 1918.

the case, but even so the displacement is more than five times the probable error. Including one more line (λ 6358), poorly determined, gives 0.53 ± 0.1 A. Using values of previous observers for eight lines we get a displacement of 0.52 ± 0.08 A. The chance that these displacements are due to accidental errors is negligible, and it would be surprising if systematic errors to the amount of 0.4 or 0.5 A should exist in either series.

The wave-lengths of four oxygen lines as found in various sources are collected in Table II. The spark lines seem to be displaced toward longer wave-lengths as compared with other sources, though by smaller amounts than occur in the similar displacements shown by the lines of hydrogen, nitrogen, and argon. The vacuum-tube values of Runge and Paschen for the second and third lines are admittedly inaccurate.

These four infra-red oxygen lines are essentially spark lines. They are faintly present in the arc[1] and are fairly strong in the solar spectrum.[2] A preliminary photograph by Mr. Ellerman and the writer indicated that the triplet at λλ 7772–7775 is much weakened in the spot spectrum as compared with the disk. This is conclusively shown by an excellent photograph secured by Mr. Brackett. It would not be surprising if this triplet were found to be strong in the spectra of certain stars of types B and A.

Argon.—The photographs show six faint lines near the positions of six of the strongest lines of the so-called "red" spectrum of argon. Lines of wave-length 4000–5000 in the "blue" argon spectrum have been previously observed in spark spectra,[3] but the writer is not familiar with any record of observation of lines of the "red" spectrum in the spark. As these lines are produced in the vacuum tube without capacity, it seems somewhat peculiar that they should appear in the condensed spark. Their effective wave-lengths in the spark appear to be greater than the vacuum-tube values.[4] The six lines give a mean displacement of 0.59±0.06 A.

Self-induction.—A few exposures were made with about 0.1 millihenrys self-induction in the electric circuit. The only striking change in the appearance of the spectrum is the pronounced weakening of the nitrogen lines λλ 6482 and 6610, though many of the lines probably become narrower.

SUMMARY

1. Fifty-eight air lines have been photographed in spark spectra from λ 5927 to λ 8719 and their wave-lengths determined with varying degrees of accuracy.

2. The lines which can be identified at present are due to argon, hydrogen, nitrogen, and oxygen.

[1] An unpublished observation by Mr. Babcock to which he kindly permits reference in this connection shows that the three lines at λλ 7772, 7774, and 7775 are quite strong at the *poles* of the iron arc, especially at the positive pole, as would be expected of spark lines.

[2] That λλ 7772, 7774, and 7775 are not caused by the earth's atmosphere, but are true solar lines, was shown by St. John (*Annual Report of the Director of Mount Wilson Observatory for 1911*).

[3] See discussion by A. S. King, *Astrophysical Journal*, 21, 344, 1905.

[4] K. W. Meissner, *Annalen der Physik*, 51, 115, 1916.

3. An attempt to distinguish between the nitrogen and oxygen lines by operating the spark in an atmosphere of oxygen confirmed several previous identifications, but was inconclusive in regard to most of the new lines.

4. The effective wave-lengths of the lines of argon, hydrogen, and nitrogen appear to be greater in the spark than in vacuum tubes by several tenths of an angstrom unit. The oxygen lines show the same effect to a smaller degree.

5. Self-induction in the spark circuit causes a relative weakening of the nitrogen lines λλ 6482.054 and 6610.39.

In conclusion it may be noted that this investigation is preliminary and incomplete. Further work on the apparent displacement of the lines is required in order that its true character and interpretation may be found. Chemical identifications should be established for the unknown lines by operating the spark in very pure atmospheres of oxygen, nitrogen, and argon.

MOUNT WILSON OBSERVATORY
February 1920

THE GOLD-POINT PALLADIUM-POINT BRIGHTNESS RATIO

By EDWARD P. HYDE and W. E. FORSYTHE

ABSTRACT

Gold-point palladium-point brightness ratio.—The authors have determined this ratio within less than 1 per cent by two sets of readings. (1) Ten determinations made with an ordinary pyrometer give $R=76.9$ for $\lambda=0.6663\,\mu$, hence $\lambda \log R=1.2566\,\mu$; (2) three determinations made with a spectral pyrometer give $R=122.2$ for $\lambda=0.6018\,\mu$, hence $\lambda \log R=1.2560\,\mu$. Taking the gold point as 1336° K, these results enable either the palladium point or c_2 to be computed if the other is known. Assuming c_2 to be $14350\,\mu$ deg., the *palladium point* comes out 1828°.5 K; while assuming the Day and Sosman value for the palladium point, 1823° K, the *radiation constant* c_2 comes out $14470\,\mu$ deg.

Double-platinum-wound black-body furnaces are described, which give a very uniform temperature and may be used at the palladium point repeatedly without appreciable deterioration.

Four quantities are related by the gold-point palladium-point brightness ratio, and if three of them are known, the fourth can be calculated. These four quantities (the two temperatures, the ratio of brightness, and c_2) are connected by the following relation from Wien's equation:

$$\log R=\frac{c_2 \log e}{\lambda}\left(\frac{1}{T_1}-\frac{1}{T_2}\right),$$

where R is the ratio of brightness for the wave-length interval whose center is at λ; T_1 and T_2 the two temperatures, on the absolute centigrade scale, in this case the gold and palladium points; and c_2 a constant of Wien's equation. Wien's equation

$$\left[E_\lambda=c_1\lambda^{-5}e^{-\frac{c_2}{\lambda T}}\right]$$

is used for such work rather than Planck's form

$$\left[E_\lambda=\frac{c_1\lambda^{-5}}{\left(e^{\frac{c_2}{\lambda T}}-1\right)}\right],$$

which has been shown by experiment to represent the facts quite well, because for the visible spectrum and not too high temperatures the two equations give practically the same values and the former is easier to use in calculation. It can be seen from the form of the two equations that the value of the product λT is the important factor in determining the difference between them. By calculation the relation shown in Table I between the two equations for different values of λT (λ expressed in μ) can be obtained.

TABLE I

λT	E_P/E_W
2000	1.0008
2500	1.003
3000	1.008
4000	1.028
5000	1.056

This table shows the ratio between Planck's and Wien's equations for different values of λT, where E_W is the brightness as determined from Wien's equation and E_P from Planck's equation. Thus, for optical pyrometry where the wave-length is always less than 0.7μ Wien's equation gives a negligibly small error over a wide range of temperature.

For two known temperatures, if the brightness ratio is measured for a particular wave-length the value of c_2 can be calculated. On the other hand, if one temperature is known, and a value for c_2 accepted, the other temperature can be calculated. The palladium melting-point in this way can be determined from the melting-point of gold and c_2. Since the value of the constant c_2 is better known than the melting-point of palladium, it seems better to use the measured ratio to determine this melting-point than to determine the value of c_2.

In the work described in this paper three different platinum-wound black-body furnaces were used. Two of these were of the same size and of the regular form with fixed diaphragms, one being made of marquardt porcelain and the other of alundum. The tubes were about 3.5 cm outside diameter and 30 cm long. The diaphragm forming the front of the uniformly heated cavity or

black body had an opening 1½ cm in diameter. This cavity as usual was located in the center of the tube. These inner heater tubes were uniformly wound with platinum ribbon, 2 cm wide and 0.01 mm thick, the distance between the different windings being as small as it was possible to wind them and avoid short circuits. This heater tube was slipped into another tube (inside diameter about 5 cm) and the space between the two tubes filled and packed tightly with very pure aluminum oxide. The aluminum oxide was held in place at the ends by two bushings made of alundum that just fitted inside of the outer tube and easily slipped over the end of the inner tube. The outside tube, which projected several centimeters beyond the inner tube, was also wound with platinum ribbon of the same size, but in this case the windings were spaced farther apart at the center of the tube in order to correct for end cooling. After a few trials a spacing for the windings was found so that with the allowable range of current through these two sets of windings the heating of the inner tube was quite uniform. The outside heater tube thus served two purposes: first to enable the platinum-wound furnace to be operated at as high a temperature as the palladium point, and, second, to correct for the end cooling. A diagram of this furnace is shown in Figure 1. The third black-body furnace was made up in the Laboratory by fitting diaphragms into a marquardt porcelain tube with an internal diameter of about 1½ cm. This heater tube was wound and mounted in exactly the same manner as the larger tubes.

Each of these furnaces contained a thermocouple inserted from the back into the central heated part in the usual manner. This was used as an indicator in holding the temperature of the furnace constant at any definite point. In this Laboratory a black-body furnace thus wound and mounted has been heated to the temperature of melting palladium as many as a dozen different times, held there for several hours each time, and is still in good working order.

The sample of gold or palladium was mounted between platinum wires supported by two porcelain tubes in such a manner that the central part of the sample could not touch either of these

two tubes. This mounting was necessary as it has sometimes been found that if a single tube is used the melted metal will touch and cling to the end of the tube, so that the electric circuit through the two platinum wires and the sample used to give a signal when the sample melted would not be broken at the instant of melting and thus a high value would be obtained. With the central part of the sample free the circuit would without doubt be broken at the instant of melting. The authors have heard it stated that this method, with the sample in the form of very fine wires, gives high values, but in this case the effect could only be differential, as

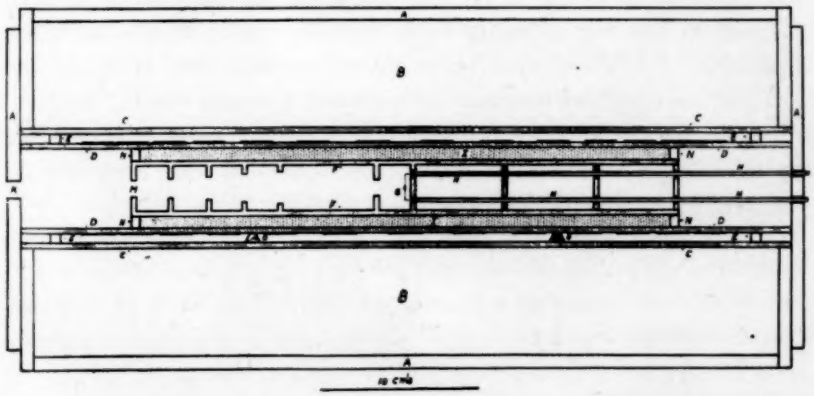

FIG. 1.—Diagram of black-body furnace tubes F and D wound with platinum ribbon, tube F uniformly and tube D about as shown to aid in correcting for end cooling. The space between these two tubes is filled and packed with very pure aluminum oxide. To prevent sagging, tube D is supported at two points, as shown, by supports O. Space outside of tubes (B) filled with infusorial earth.

the same method was used for each metal. Furthermore, as the samples were mounted in several different ways and no difference was detected, it is thought that such errors, if they exist, are very small when care is used in the mounting. Two sets of samples of both gold and palladium were used. They were obtained from the American Platinum Works at different times, about two and one-half years apart. In ordering these samples it was specified that the purest Heraeus metals for melting-point determinations were to be furnished. No differences could be detected between the two samples of gold or the two samples of palladium.

Values of this ratio were measured by two different forms of pyrometer; first, the ordinary pyrometer where a red glass is used as the monochromatic screen, and, second, a spectral pyrometer.

When values of the ratio of brightnesses of a black body between two temperatures are measured, using an optical pyrometer, the wave-length is determined from the effective wave-length of the red glass used in the eyepiece. In using the ordinary optical pyrometer, it is the integral luminosities through the red glass screen that are compared, for which reason the effective wave-length of red glass screen for a certain temperature interval has been defined as the wave-length, such that the ratio of its radiation intensities for these two temperatures equals the ratio of the integral luminosities through the particular screen used.

Knowing the spectral transmission of the red glass, it is possible to calculate the effective wave-length, λe, for the temperature interval by means of the following equations:

$$\left[\frac{J(\lambda T_1)}{J(\lambda T_2)}\right]_{\lambda e} = \frac{\int_0^\infty J(\lambda T_1) t'_R V_\lambda d\lambda}{\int_0^\infty J(\lambda T_2) t'_R V_\lambda d\lambda}$$

where $J(\lambda T)d\lambda$ is the energy as given by Wien's equation for the wave-length interval from λ to $\lambda+d\lambda$; t'_R is the spectral transmission of the red glass; and V_λ is the visibility. These integrals can be computed by the step-by-step method (each step $0.005\,\mu$) with sufficient accuracy for this purpose. To determine the temperature of the palladium point to better than one degree requires that the wave-length be known a little better than one part in six hundred. By the foregoing method of calculation the effective wave-length can be determined to about one or two parts in six thousand.

By the use of a spectral pyrometer a determination of this brightness ratio can be made, and, at the same time, if proper precautions are taken, the wave-length easily determined. This instrument differs from the ordinary pyrometer in that a spectro-

scope is used in place of the ordinary eyepiece. If, with this spectral pyrometer, care is taken to correct for the slit-widths in the regular manner, or to avoid such corrections by using very narrow slits, both for the collimator and the eyepiece, the wavelength used will be very definitely known. In the spectral pyrometer used to determine this ratio, a Hilger constant-deviation spectroscope was used for the eyepiece and such values used for the slit-widths (each slit 0.3 mm, which transmitted a range of wavelength amounting to about $0.005\,\mu$ for $\lambda = 0.6\,\mu$) that the error due to this source was negligible.

When making the foregoing determinations of this brightness ratio with either pyrometer, the brightness for each melt was measured in terms of the current through the pyrometer filament. The sample was melted inside the black-body cavity of the furnace. The temperature of the latter was raised very slowly and the E.M.F. of the thermocouple very carefully followed by the observers and thus the values were obtained at the exact time of melt. By the use of an electrical bell which rang when the sample melted, the observer was able to give his attention to following the changes in the E.M.F. of the thermocouple and at the same time to be sure when the sample melted. A very high resistance relay was inserted in the circuit so as to avoid heating the sample with the current used. The E.M.F. of the thermocouple at the time of melt was employed as an indicator and thus the furnace was held at this temperature as long as was necessary to measure this brightness with the pyrometer. In this manner several melts were made and the brightness measured with one metal and then the temperature of the furnace was changed and several melts were made and the brightness also measured with the other metal.

When the black body was at the temperature of the palladium point proper sector disks were used to give an apparent brightness just above and just below that of the black body at the temperature of melting gold. By interpolation, using the logarithms of the current and the relative brightness as obtained from the reading with these and other sectors (since the logarithms of current and brightness give nearly a linear relation), the value of the ratio corresponding exactly to the gold-point brightness was found.

The sectors used had been very carefully calibrated in this Laboratory by a photometric method by Mr. Cady. The six-degree sector used was calibrated on a circular dividing engine by the Bureau of Standards and a value found that agreed very closely with that found by the method used in this Laboratory.

In Table II are given the results for the two different pyrometers. The values for the palladium point were calculated from the ratios found from the value of 1336° K for the gold point and from $c_2 = 14350\,\mu$ deg. The range of values of the ratio found by means of the ordinary optical pyrometer is from 1 per cent above to the same amount below the mean value given in Table II. This range in the ratio corresponds to about $\frac{1}{12}$ of 1 per cent in the value for the melting-point of palladium. Thus the range due to this error is about $\pm 1.5°$ K. The range for the value using the spectral pyrometer was about the same, being in this case somewhat less than 1 per cent. As a result of this work the laboratories of the General Electric Company have decided, for the time being, to accept 1828° K as the palladium point.

TABLE II

SUMMARY OF RESULTS OF MEASUREMENTS OF THE GOLD-POINT PALLADIUM-POINT BRIGHTNESS RATIO

METHOD	NUMBER OF SEPARATE DETERMINATIONS	NUMBER OF MELTS		λ	AVERAGE OF RATIO FOUND	RESULT FOR THE PALLADIUM POINT
		Pd.	Au.			
Ordinary Pyrometer*....	10	45	45	0.6663	76.9	1828°.6 K
Spectral Pyrometer.....	3	13	12	0.6018	122.2	1828°.3 K

*A statement of these results was published several years ago (*Astrophysical Journal*, 42, 300, 1915), but at that time the values were used to determine c_2 rather than the palladium point.

The value found for the melting-point of palladium from the measured ratio and the value accepted for the value of c_2 is about 5° C. higher than the value given by Day and Sosman in their extended work with the gas thermometer. These experimenters allow an error of $\pm 2°$ C. There has been no chemical analysis of the palladium used in this work but it is doubtful if this is the cause

of the difference. More extended work should be done in order to settle this point. No attempt has been made to refer to all the work that has been done on this point. An excellent review of the field is found in a paper by Waidner and others in the *Report of the Symposium on Pyrometry* soon to be issued by the American Institute of Mining and Metallurgical Engineers.

NELA RESEARCH LABORATORY, CLEVELAND, OHIO
March 1920

Minor Contributions and Notes

FIVE Oe5 STARS WITH VARIABLE RADIAL VELOCITIES

ABSTRACT

Class Oe5 stars with variable radial velocities.—As in the case of most of the stars of this spectral class which have been investigated, it has been found that *9 Sagittae*, *A Cygni*, *B.D.+37°1146*, *B.D.+44°3639*, and *B.D.+56°2617* have variable velocities and therefore probably belong to binary or multiple systems. Measurements of twenty-nine plates, scattered over a period of several years, are given in a table and show variations for the different stars amounting to from 20 to 160 km/sec., variations which are from 8 to 60 times the probable error. In addition, variations of about 10 km/sec. were found for *B.D+52°726* and *19 Cephei*.

During the progress of a radial velocity program at the Detroit Observatory of stars of spectral class Oe5 the following were found by the writer to have variable velocities. The table is self-explanatory. The radial velocity is reduced to the sun.

The lines measured for the radial velocity determination include hydrogen β, γ, δ, ϵ, ζ, the ζ Puppis series γ', δ', ϵ', ζ', helium $\lambda\lambda$ 4713, 4388, 4144, 4026, 4009, and the four lines whose wavelengths were determined by Frost[1] in 10 Lacertae, a star of the same spectral class, $\lambda\lambda$ 4685.90, 4116.33, 4097.55, 4089.12. The H and K lines of calcium were also measured, but in general their range of displacement is small.

The width of the hydrogen lines, the diffuse character of the ζ Puppis series, changes in the relative intensities of various lines, and apparent shifts in the center of absorption make the determination of velocity rather difficult. Some of the lines of the third plate of A Cygni appeared to be doubled, but consistent velocities could not be obtained. The radial velocities referred to the sun have been determined to the nearest tenth of a kilometer per second, although the probable error for a single plate based upon the agreement of the lines used is about two or three kilometers. Two

[1] *Astrophysical Journal*, 40, 268, 1914.

other stars of the program, B.D.+52°726 and 19 Cephei, gave ranges of ten kilometers on five plates each, only two kilometers less than the range in the case of 10 Lacertae.

Including six other stars of class Oe5 having variable radial velocities, ξ Persei, θ^1 Orionis (C), ι Orionis, S Monocerotis, τ Canis

Star Position 1900 Phot. Magnitude	Date	Julian Day	Radial Velocity	Quality of Plate
		242	km	
B.D.+37°1146	Oct. 13, 1917	1,515.824	− 2.3	Weak
	Dec. 11, 1917	574.755	−20.3	Fair
5^h14^m0	Feb. 1, 1919	991.733	− 2.8	Fair
+37°20′	Feb. 5, 1919	995.653	− 1.8	Fair
6.71	Feb. 18, 1919	2,008.599	+ 0.3	Fair
	Mar. 27, 1920	2,411.656	− 8.9	Weak
	Mar. 29, 1920	2,413.635	−10.4	Fair
9 Sagittae	June 18, 1918	1,763.760	+ 9.7	Good
	July 8, 1918	783.674	+17.1	Fair
19^h47^m9	July 12, 1918	787.764	− 7.9	Fair
+18°25′	Aug. 3, 1918	809.726	− 2.1	Fair
6.29	Aug. 5, 1918	811.692	− 2.5	Fair
B.D.+44°3639	July 19, 1918	1,794.762	−17.5	Fair
	July 25, 1918	800.736	−12.9	Fair
20^h53^m1	Aug. 2, 1918	808.726	+22.2	Weak
+44°33′	Aug. 5, 1918	811.762	+ 9.2	Good
6.01	Aug. 9, 1918	815.687	+16.4	Fair
	Aug. 10, 1918	816.740	+17.1	Fair
A Cygni	Nov. 8, 1915	0,810.635	+28.8	Fair
	Aug. 3, 1919	1,809.793	−27.9	Fair
21^h14^m8	Aug. 5, 1918	811.816	?	Fair
+43°31′	Aug. 7, 1918	813.683	+ 7.	Poor
6.06	Aug. 12, 1918	818.703	−54.0	Fair
B.D.+56°2617	Nov. 9, 1915	0,811.600	+92.6	Good
	July 20, 1918	1,795.858	−57.2	Fair
21^h35^m9	July 25, 1918	800.812	−30.4	Fair
+57°2′	Aug. 7, 1918	813.753	+56.5	Poor
5.64	Aug. 10, 1918	816.792	+87.5	Fair
	Aug. 12, 1918	818.758	−65.2	Fair

Majoris, and 10 Lacertae, it appears that a very large proportion of the stars of this class that have been investigated have variable velocities; and a star of class Oe5 with a constant velocity may prove to be the exception rather than the rule.

It is interesting to note also that eight out of fourteen Oe5 stars here observed belong to well-known visual binary or multiple systems, λ Orionis, θ^1 Orionis, δ Orionis, S Monocerotis, τ Canis

Majoris, B.D.+56°2617, 19 Cephei, and 10 Lacertae. Including the two mentioned above with velocity ranges of ten kilometers and interpreting variable radial velocity stars as spectroscopic binaries, the entire number, fourteen, belong to binary or multiple systems.

W. CARL RUFUS

DETROIT OBSERVATORY
ANN ARBOR, MICH.
April 16, 1920

THE PARALLAX OF THE B-STAR BOSS 1517

ABSTRACT

The parallax of the B-Star Boss 1517 has been recomputed. The result is

$$\pi = +0''.048 \pm 0''.021.$$

In the *Observatory* for July 1919 (**42**, 281) Mr. A. J. Roy gives $-2''.3$ as the corrected Boss proper motion in right ascension for the B-star Boss 1517. With this proper motion I have recomputed my parallax plates and get

$$\pi = +0''.048 \pm 0''.021.$$

This is still a large parallax for a star of spectral class B and in better agreement with the hypothetical parallax of $0''.033$ computed by Professor Kapteyn (*Astrophysical Journal*, **47**, 104, 1918).

J. VOÛTE

MET. OBSERVATORIUM
WELTEVREDEN, JAVA
February 14, 1920

PREPARATION OF ABSTRACTS

Every article in the *Astrophysical Journal*, however short, is to be preceded by an abstract prepared by the author and submitted by him with the manuscript. The abstract is intended to serve as an aid to the reader by furnishing an index and brief summary or preliminary survey of the contents of the article; it should also be suitable for reprinting in an abstract journal so as to make a reabstracting of the article unnecessary. Therefore, *the abstract should summarize the information completely and precisely*, and also, in order to enable a reader to tell at a glance what the article is about and to enable an efficient index of the subject-matter of the abstract to be readily prepared, *the abstract should contain a set of subtitles which together form a complete and precise index of the information contained in the article.*

In the preparation of abstracts, authors should be guided by the following rules, which are illustrated by the abstracts appearing in the *Astrophysical Journal* for January and March 1920.[1] First the new information contained in the article should be determined by a careful analysis; then the subtitles should be formulated; and finally the text should be written and checked.

RULES

1. *Material not new* need not be analyzed or described; a valuable summary of previous work, however, should be noted.

2. *The subtitles should together include all the new information;* that is, every measurement, observation, method, improvement of apparatus, suggestion, and theory which is presented by the author as new and of value in itself.

3. *Each subtitle should describe the corresponding information so precisely* that the chance of any investigator's being misled into thinking the article contains the particular information he desires when it does not, or vice versa, may be small. "Zeeman effect for metallic furnace spectra" is too broad unless all metals have been studied, for an investigator may be interested, at the time, in only one metal; but "Infra-red arc spectrum of iron to 3μ" evidently satisfies this rule. It is particularly desirable that ranges of variation of temperature, wave-length, pressure, etc., be given.

4. *The text should summarize the author's conclusions and should transcribe all numerical results of general interest*, including all that might be looked for in a table of astronomical and physical constants, with an indication of the

[1] The rules and illustrative abstracts were prepared by G. S. Fulcher, of the National Research Council.

accuracy of each. It should give all the information that anyone, not a specialist in the particular field involved, might care to have in his notebook.

5. *The text should be divided into as many paragraphs* as there are distinct subjects concerning which information is given. Parts of subtitles may be scattered through the text but the subject of each paragraph must be indicated at the beginning.

6. *Complete sentences* should be used except in the case of subtitles. The abstract should be made as readable as the necessary brevity will permit.